$$Z(E) = 2\frac{dn}{dE} = \frac{8mL^2}{h^2}\frac{1}{n} \quad \text{--- (4)}$$

As $E_n = \frac{\hbar^2 k^2}{2m}$

$$= \left(\frac{h}{2\pi}\right)^2 \left(\frac{n\pi}{L}\right)^2 / 2m$$

$$E_n = \frac{h^2 n^2}{8mL^2}$$

or $n^2 = \frac{8mL^2 E_n}{h^2}$ | $E = E_n$

or $n = \sqrt{8mE}\,\frac{L}{h}$

or $\frac{1}{n} = \frac{h}{L\sqrt{8mE}}$ --- (5)

(4) & (5) $Z(E) = \frac{8mL^2}{h^2} \times \frac{h}{L\sqrt{8mE}}$

$$= \frac{L\sqrt{8m}}{h\sqrt{E}}$$

$$\boxed{Z(E) = \frac{4L}{h}\left(\frac{m}{2E}\right)^{1/2}} \quad \text{--- (6)}$$

equ^n (6) represent the density of states in 1-Dimensional case.

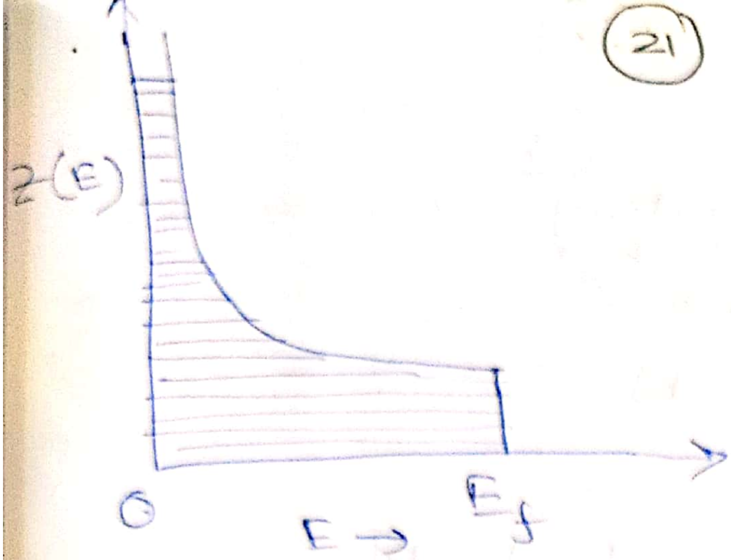

Fig shows that levels are filled from E = 0 to $E = E_f$ and beyond E_f all levels are vacant. Fermi level divides the filled and unfilled levels in the metals at absolute zero.

Average k.E in the ground state

The ~~total~~ eng E_0 of all N electrons in ground state is given by

$$E_0 = 2 \sum_{n=1}^{N/2} E_n \quad \text{--- (1)}$$

To accumodate N electrons we need N/2 eng levels. In one eng level we can accomodate 2 electrons with equal eng.

As

$$E_0 = 2 \frac{\hbar^2}{2m} \left(\frac{n\pi}{L}\right)^2$$

$$= 2 \frac{\hbar^2}{2m} \left(\frac{\pi}{L}\right)^2 \sum_{n=1}^{N/2} n^2$$

as $\displaystyle\sum_{n=1}^{N/2} n^2 = \frac{1}{6} s(2s^2+3s+1)$

$$\simeq \frac{1}{3} s^3 \text{ for } s \gg 1$$

$$\simeq \frac{1}{3} \left(\frac{N}{2}\right)^3 \quad \text{--- (2)}$$

$s = N/2$

From 1 & 2

$$E_0 = 2\frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 \times \frac{1}{3}\left(\frac{N}{2}\right)^3$$

$$= \frac{1}{3}\frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2 N$$

$$\cancel{= \frac{1}{3} N E_f \left(\frac{h}{2\pi}\right)^2 \frac{1}{2m}\left(\frac{N\pi}{2L}\right)^2 N}$$

$$\cancel{= \frac{1}{3} N}$$

$$E_0 = \frac{1}{3} N E_f \qquad \left[E_f = \frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2 \right]$$

∴ Average K.E in the ground state of electron confined to 1-D box is

$$\langle E_0 \rangle = \frac{E_0}{N}$$

$$\boxed{\langle E_0 \rangle = \frac{1}{3} E_f}$$