The allowed wave f^n $\psi_n(x)$ & Eng E_n exist for integral values of n, called the quantum number. Thus eng spectrum of the electrons are discrete in nature. If L is (L≈1cm) large in eqn (6) the adjacent permitted energy levels differ by about 10^{-19} eV. This means that the eng levels form a quasicontinous band.

Evaluation of A in wave f^n

$$\psi(x) = A\sin\left(\frac{n\pi}{L}\right)x$$

Since the probability of finding the electron anywhere within $x=0$ & $x=L$ is unity,

$$\therefore \int_0^L \psi_n^*(x)\,\psi_n(x)\,dx = 1$$

$$\text{or} \int_0^L A^2 \sin^2\left(\frac{n\pi}{L}\right)x\, dx = 1$$

$$\text{or}\ \frac{A^2}{2}\int_0^L \Big[1 - \underbrace{\cos\left(\frac{2\pi n}{L}x\right)}_{0}\Big]\,dx = 1$$

or $\frac{A^2}{2}\int_0^L dx = 1$

or $\frac{A^2}{2}\left[x\right]_0^L = 1$ $\quad \therefore \frac{A^2}{2}L = 1$

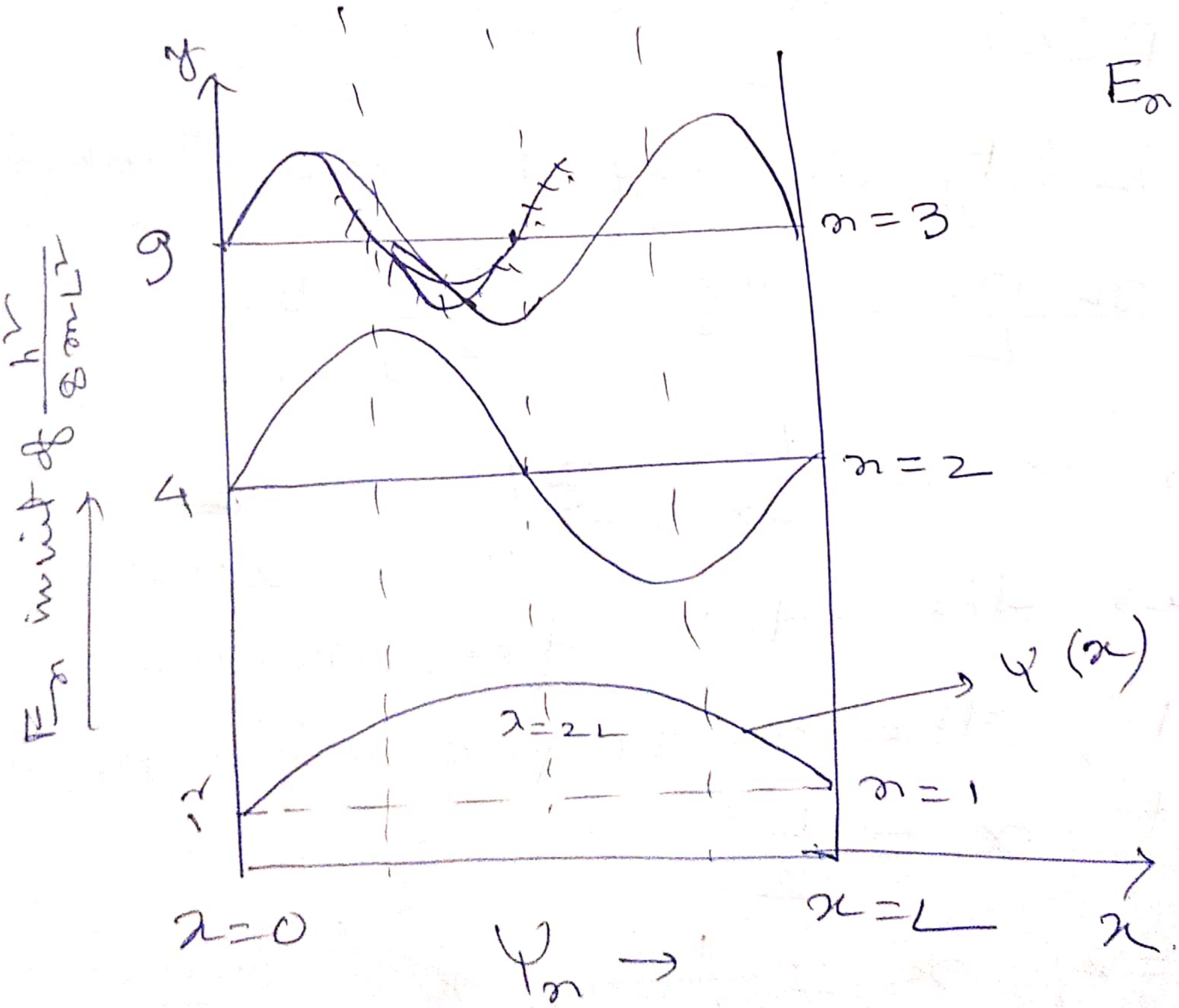
or $A = \sqrt{\frac{2}{L}}$

$\therefore \psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)} \quad \text{—— (7)}$$

The wavefn's can be drawn for $n = 1, 2, 3.$

$E_n = \frac{n^2 h^2}{8mL^2}$



Probability of finding a particle

The probability of finding a particle over a small distance dx at x is $p(x)\,dx = |\psi_n|^2 dx$

$$p(x)\,dx = \frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right)dx$$

The probability density for the one dimensional motion is

$$p(x) = \frac{2}{L}\sin^2\frac{n\pi x}{L}$$

Probability density is max^m when

$$\frac{n\pi x}{L} = \pi/2,\ \frac{3\pi}{2},\ \frac{5\pi}{2}$$

$$\text{or } x = \frac{L}{2n},\ \frac{3L}{2n},\ \frac{5L}{2n}$$

Thus the state for which $n=1$, the most probable position is at $x = \frac{L}{2}$

For $n=2$, $x = \frac{L}{4}$, $x = \frac{3L}{4}$.

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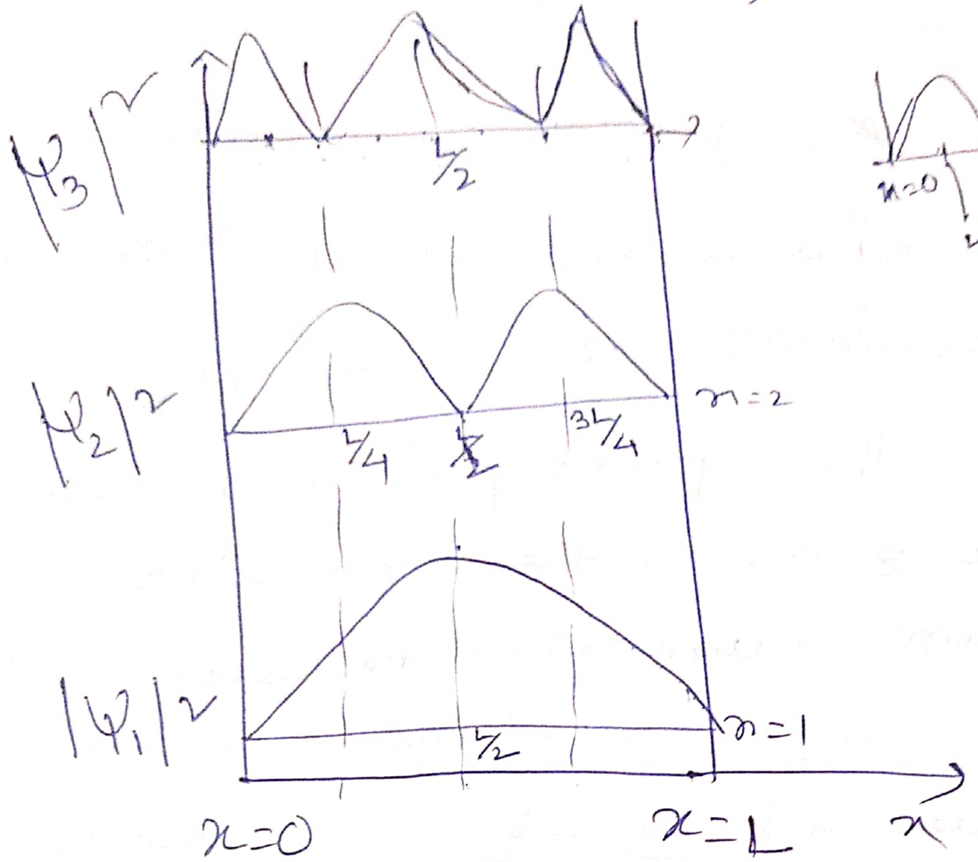
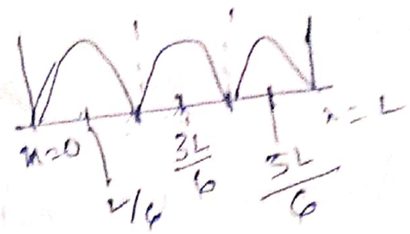
For $n = 3$, $x = \frac{L}{6}$, $x = \frac{3L}{6}$, $x = \frac{5L}{6}$





Thus the wave fn and probability-density fn for the particle in a one dimensional box can be written as

$$\psi_1(x)_{n=1} = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}, \quad P_1(x) = |\psi_1(x)|^2 = \frac{2}{L}\sin^2\frac{\pi x}{L}$$

$$\psi_2(x)_{n=2} = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}, \quad P_2(x) = |\psi_2(x)|^2 = \frac{2}{L}\sin^2\frac{2\pi x}{L}$$

Eng values are

$$E_1 = \frac{n^2h^2}{8mL^2} = \frac{1^2 h^2}{8mL^2}$$

$$E_2 = 2^2\frac{h^2}{8mL^2} \quad - - - - \quad E_n = .38 n^2 \text{ eV}$$

$$E_1 = 38 \text{ eV}$$

$$\boxed{E_n = \frac{n^2h^2}{8mL^2}}$$

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