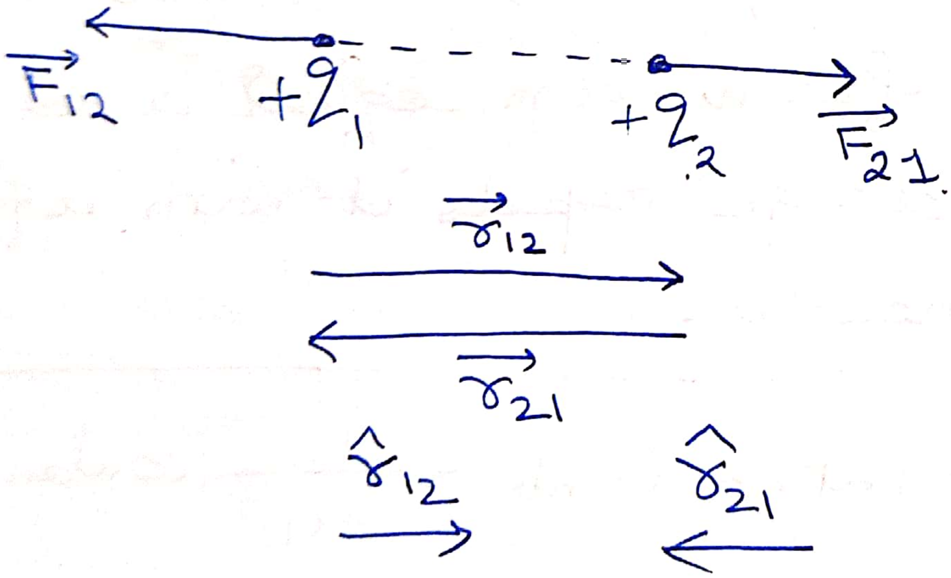
Coulomb's law in vector form

(i) When both the charges are of similar nature (like charges) ie $q_1 q_2 > 0$

So repulsive force between two "+ve" charges and two "−ve" charges can be written in vector form as



$$\vec{F}_{12} = K \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

In air medium $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cancel{K} \frac{q_1 q_2}{r^2} \hat{r}_{21}$

where $r = |\vec{r}_{12}| = |\vec{r}_{21}|$ —— (2)

and $\vec{r}_{21} = |r_{21}|\, \hat{r}_{21}$

or $\hat{r}_{21} = \dfrac{\vec{r}_{21}}{|r_{21}|}$ —— (3)

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \frac{\vec{r}_{21}}{|r_{21}|}$$

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21}}$$ using (2) —— (4)

Similarly $\vec{F}_{21} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q_1 q_2}{r^2} \hat{r}_{12}$

or $\vec{F}_{21} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q_1 q_2}{r^2} \dfrac{\vec{r}_{12}}{|r_{12}|}$

or $$\boxed{\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12}}$$ —— (5)

using (2)

As $\vec{r}_{21} = -\vec{r}_{12}$, so from equation (4) & (5) we can write

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$ —— (6)

Here

$\vec{F_{12}}$ = Force acting on charge q_1 due to q_2

$\vec{F_{21}}$ = Force acting on charge q_2 due to q_1

$\vec{r_{12}}$ = Displacement from charge q_1 to

$\vec{r}_{21}$ = Displacement from charge q_2 to q

$\hat{r}_{12}$ = Unit vector in the direction of $\vec{r}_{12}$

$\hat{r}_{21}$ = Unit vector in the direction of $\vec{r}_{21}$

(ii) When the two charges are of different nature (unlike charge) $(q_1 q_2 < 0)$, so attractive forces between ~~to~~ one '+ve' charge and one "−ve" charge can be written in vector form as.

$$\vec{F_{12}} = \cancel{K}\frac{q_1 q_2}{r^2}\,\hat{r}_{12}$$

In air medium $K = \frac{1}{4\pi\epsilon_0}$ (S.I unit)

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \text{—— (1)}$$

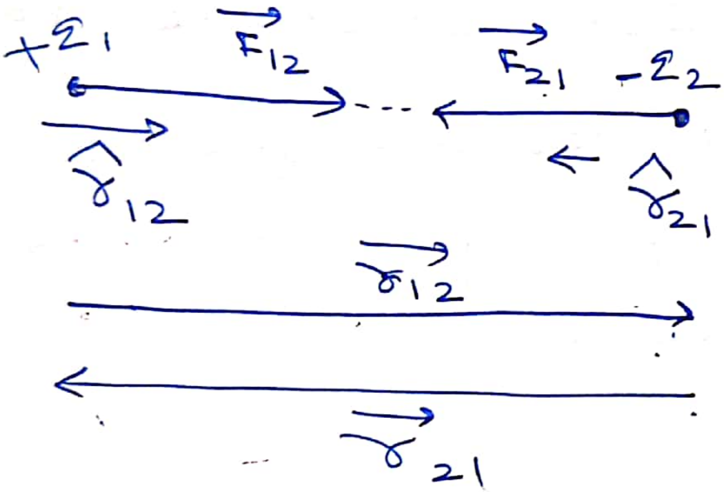
Here $\vec{F}_{12}$ and $\vec{r}_{12}$ are directed in same direction.

$\vec{r} = |r| \hat{r}$

$$\text{or } \vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \frac{\vec{r}_{12}}{|r_{12}|}$$

$$\text{or } \boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12}} \quad \text{—— (2)}$$

As $|r_{12}| = r$



Similarly attractive force on charge q_2 due to q_1 we can write

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \frac{\vec{r}_{21}}{|r_{21}|}$$