7. Drug assimilation model :→

We investigate two simple models of cold pill assimilation into the bloodstream that are adopted from Borelli and Coleman (1996). In the first model we consider a single cold pill and in the second model a course of cold pills.

We can consider drug assimilation model as a compartmental model with two compartments, corresponding to the GI-tract (Gastro-Intestinal tract) and the bloodstream. The GI-tract compartment has a single input and output and the bloodstream compartment has a single input and output.

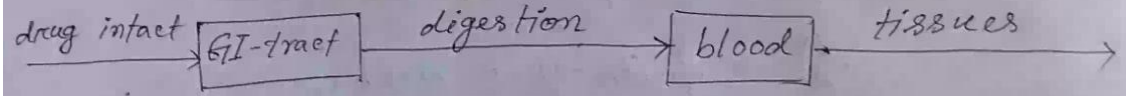

fig.: Input output compartmental diagram for drug ~~assig~~ assimulation.

Using Balance law the word eqn are given by,

$$\left\{\begin{array}{l}\text{rate of change}\\ \text{of drug}\\ \text{in GI-tract}\end{array}\right\} = \left\{\begin{array}{l}\text{rate of}\\ \text{drug}\\ \text{intake}\end{array}\right\} = \left\{\begin{array}{l}\text{rate of}\\ \text{drug leaves}\\ \text{GI-tract}\end{array}\right\}$$

and

$$\left\{\begin{array}{l}\text{rate of change}\\ \text{of drug}\\ \text{in blood}\end{array}\right\} = \left\{\begin{array}{l}\text{rate of}\\ \text{drug}\\ \text{enters blood}\end{array}\right\} = \left\{\begin{array}{l}\text{rate of}\\ \text{drug leaves}\\ \text{blood}\end{array}\right\}$$

We consider two models :

1. A single ~~code~~ cole pill where there is no ingestion of the drug except that which occurs initially.

2. A course of cold pill where the drug intake is assumed to occur continuously.

Model 1 : A single cold pill.

In GI-tract, we considered the pill to have swallowed the pill dissolved and the drug begins to enter the bloodstream from GI-tract. So, for GI-tract there is only an output term.

Let, $u(t)$ & $v(t)$ be the amount of a drug in the GI-tract and in the blood stream at time t respectively.

We assume the output rate ∝ GI-tract drug concentration, which is proportional to the amount of drug in the blood stream. Then by balance law, we have—

$$\frac{du}{dt} = -m_1 u; \quad u(0) = u_0 \qquad \text{——} (1).$$

where, u_0 is the amount of a drug in the pill, our initial cond^n and m_1 is the rate constant.

The pill dissolved and diffuse into the bloodstream from the GI-tract. In the bloodstream, $v(0) = 0$. The level increase as drug diffuse from GI-tract and decreases as kidneys and liver remove it.

Using balance law, we have—

$$\frac{dv}{dt} = m_1 u - m_2 v; \quad v(0) = 0 \qquad \text{——} (2).$$

where $m_1 \neq m_2$.

From (i), $\frac{du}{u} = -m_1 dt$.

$\Rightarrow \log u = -m_1 t + A$.

$\Rightarrow u(t) = e^{-m_1 t + A} = e^A \cdot e^{-m_1 t}$.

using $u(0) = u_0$, we get, $e^A = u_0$.

Thus, $u(t) = u_0 e^{-m_1 t}$.

From (ii), $\frac{dv}{dt} = m_1 u_0 e^{-m_1 t} - m_2 v$.

$\Rightarrow \frac{dv}{dt} + m_2 v = m_1 u_0 e^{-m_1 t}$.

which is a linear differential eqn.

I.F. $= e^{\int m_2 dt} = e^{m_2 t}$

$\therefore v \cdot (IF) = \int (IF)\, m_1 u_0 e^{-m_1 t} dt + c$.

$\Rightarrow v e^{m_2 t} = \int e^{m_2 t} \cdot m_1 u_0 e^{-m_1 t} dt + c$.

$\Rightarrow v e^{m_2 t} = m_1 u_0 \frac{e^{(m_2 - m_1)t}}{(m_2 - m_1)} + c$.

using, $v(0) = 0$, we get, $c = -\frac{m_1 u_0}{m_2 - m_1}$.

$\therefore v e^{m_2 t} = m_1 u_0 \frac{e^{(m_2 - m_1)t}}{m_2 - m_1} - \frac{m_1 u_0}{m_2 - m_1}$.

$\Rightarrow v e^{m_2 t} = \frac{m_1 u_0}{m_2 - m_1} \left[e^{(m_2 - m_1)t} - 1 \right]$

$\Rightarrow v = \frac{m_1 u_0}{m_2 - m_1} \left[e^{-m_1 t} - e^{-m_2 t} \right]$

As 't' increases, both 'u' and 'v' approach to '0'. Although the rate at which this occurs, depends on the coefficients m_1 and m_2 associated with each other.

Model 2 : A course of cold pill

We assume that the drug is delivered to the GI-tract continuously, which is reasonable for pills that dissolve slowly in the GI-tract and a constant rate of drug input I.

Since the pill dissolve slowly therefore initially, there is no drug in the GI-tract.

Let, $u(t)$ and $v(t)$ are the amount of drug in the GI-tract and in bloodstream at time t respectively. By balance law, word eq^{n}s are given by,

$$\frac{du}{dt} = I - m_1 u \; ; \quad u(0) = 0 \quad \text{——(i)}.$$

$$\frac{dv}{dt} = m_1 u - m_2 v \; ; \quad v(0) = 0 \quad \text{——(ii)}.$$

Where I is a positive constant representing the rate of ingestion of the drug.

Note: Here $u(0) = 0$, whereas in the previous model $u(0) = u_0$.

From $\frac{du}{dt} = I - m_1 u$.

$$\Rightarrow \frac{du}{dt} + m_1 u = I.$$

$$IF = e^{\int m_1 dt} = e^{m_1 t}$$

$$\therefore u e^{m_1 t} = \int e^{m_1 t} \cdot I \, dt + c.$$

$$\Rightarrow u e^{m_1 t} = \frac{e^{m_1 t}}{m_1} I + c \quad \text{——(iii)}.$$

using $u(0) = 0$, we get $c = -\frac{I}{m_1}$.

$$\therefore u e^{m_1 t} = \frac{I}{m_1} e^{m_1 t} - \frac{I}{m_1}$$

$$\Rightarrow u = \frac{I}{m_1}\left(1 - e^{-m_1 t}\right)$$

putting the value of u in eqn (ii),

$$\frac{dv}{dt} = m_1 \frac{I}{m_1}\left(1 - e^{-m_1 t}\right) - m_2 v.$$

$$\Rightarrow \frac{dv}{dt} + m_2 v = I\left(1 - e^{-m_1 t}\right).$$

which is a linear d. eqn.

$$IF = e^{\int m_2 dt} = e^{m_2 t}.$$

$$\therefore v \cdot e^{m_2 t} = \int e^{m_2 t} \cdot I\left(1 - e^{-m_1 t}\right) dt + c_1.$$

$$\Rightarrow v e^{m_2 t} = I\left[\frac{e^{m_2 t}}{m_2} - \frac{e^{(m_2 - m_1)t}}{m_2 - m_1}\right] + c_1$$

using, $v(0) = 0$, we get

$$c_1 = -I\left[\frac{1}{m_2} - \frac{1}{m_2 - m_1}\right]$$

Therefore,

$$v(t) = I\left[\frac{1}{m_2} - \frac{e^{-m_1 t}}{m_2 - m_1}\right] - I\left[\frac{1}{m_2} - \frac{1}{m_2 - m_1}\right] e^{-m_2 t}$$

$$\Rightarrow v(t) = \frac{I}{m_2}\left[1 - \frac{1}{m_2 - m_1}\left(m_2 e^{-m_1 t} - m_1 e^{-m_2 t}\right)\right]$$