4. <u>Seasonal Flow Rate</u>:- Suppose that the flow rate in and out of lake $f(t)$ varied seasonally.

let, $\sin(2\pi t) < 1$ and ϵ is very small.

let $f(t) = f_0(1+\epsilon\cos 2\pi t)$, where f_0 is the mean in/out flow (with units of litres per year) and $0<\epsilon<1$ so that $f(t) \geq 0$.

Here $2f_0\epsilon$ is maximum variation in the flow rate and we assume that to be small compared to the mean flow rate f_0.

We know, $$\frac{dc}{dt} = \frac{f}{V}c_{in} - \frac{f}{V}c$$

If $c_{in}=0$, then $\frac{dc}{dt} = -\frac{f}{V}c$.

$$\Rightarrow \frac{dc}{dt} = -\frac{f_0(1+\epsilon\cos 2\pi t)}{V}c.$$

$$\Rightarrow \frac{dc}{c} = -\frac{f_0}{V}(1+\epsilon\cos 2\pi t)\,dt.$$

Integrating, we have—

$$\log c = -\frac{f_0}{V}\left(t+\frac{\epsilon\sin 2\pi t}{2\pi}\right) + \cancel{\cos}\, A.$$

$$\Rightarrow c = e^{-\frac{f_0}{V}\left(t+\frac{\epsilon\sin 2\pi t}{2\pi}\right)+A}.$$

$$\Rightarrow c = e^{A}e^{-\frac{f_0}{V}\left(t+\frac{\epsilon\sin 2\pi t}{2\pi}\right)}$$

Using, $c(0)=c_0$, we get $c_0 = e^{A}$.

Therefore, $$c = c_0 e^{-f_0\left[t+\frac{\epsilon}{2\pi}\sin 2\pi t\right]/V}.$$

Since, $\cancel{\epsilon}\sin 2\pi t \leq 1$ so, $\frac{\epsilon}{2\pi}\sin 2\pi t \leq \frac{\epsilon}{2\pi}$.

$$\Rightarrow \frac{f_0}{V}\left[t+\frac{\epsilon}{2\pi}\sin 2\pi t\right] \leq \frac{f_0}{V}\left[t+\frac{\epsilon}{2\pi}\right].$$

$$\Rightarrow c_0 e^{-\frac{f_0}{V}\left[t+\frac{\epsilon}{2\pi}\sin 2\pi t\right]} \leq c_0 e^{-\frac{f_0}{V}\left[t+\frac{\epsilon}{2\pi}\right]}$$

$$\Rightarrow c(t) \leq c_0 e^{-\frac{f_0}{V}\left[t+\frac{\epsilon}{2\pi}\right]}.$$

$\Rightarrow t = \frac{V}{f} \log 20 \simeq \frac{3V}{f}$.

Lake Erie :→ Given that $V = 458 \times 10^9 m^3$, $f = 480 \times 10^6 m^3/day$

$c(t) = 5\%$ of $c_0 = 0.05 c_0$.

Putting these values in $c(t) = c_0 e^{-\frac{f}{V}t}$, we get —.

$$0.05 c_0 = c_0 e^{-\frac{480 \times 10^6}{458 \times 10^9} t}$$

$$\Rightarrow 0.05 = e^{-\frac{480}{458 \times 10^3} t}.$$

$$\Rightarrow \log 0.05 = -\frac{480}{458 \times 10^3} t$$

$$\Rightarrow t = \frac{458 \times 10^3}{480} \times \log 20$$

$$= 2862.5 \text{ days}.$$

6. Case study (Lake Burley Griffin) :-

The information from this case study is adopted from the research paper Burges and Olive (1975). Lake Burley Griffin in Canberra, the capital city of Australia, was created artificially in 1962 for both recreational and aesthetic purposes. In 1974, the public health authorities indicated that pollution standards set down for safe recreational use were being violated and that this was attributable to the sewage works in Queanbeyan upstream.

After extensive measurements of pollution level taken in the 1970s it was established that, while the seaways plants (of which there are three above the lake) certainly exacerbated the problem, there were significant contributions from rural and urban runoff as well, particularly during summer rainstorms. These contributed to dramatic increases in pollution levels and at time were totally responsible for lifting the concentration levels above the safety limit. As a point of interest Queanbeyan (where the sewage plants are situated) is in the state of New South Wales (NSW), while the lake is in the Australia capital territory, and although they are a ten minute drive apart the safely levels/ standards for those who you swim in NSW are different from the standards for those who swim in the capital territory.

<u>Example:-</u> In 1974 the mean concentration of the bacteria Faecal coliform count was approximately 10^7 bacteria per m^3 at the point where the rivers feeds into the lake. The safety threshold for this Faecal coliform count in the water is such that for contact recreational sports no more than 10% of total samples over a 30 days period should exceed 4×10^6 bacteria per m^3.

Given that the lake was polluted it is of interest to examine how, if sewage management were Improved. The lake would flush out and if and when the pollution level would drop below the safety threshold.

Solution :- Let us assume the following for the given system :

(i) flow v into the lake is assumed equal to flow out of the lake.

(ii) Volume V of the lake is constant and is approximately $28 \times 10^6 \ m^3$.

(iii) The lake is well mixed in the sense ~~of~~ that the pollution concentration throughout will be taken as constant.

Under the following assumptions, the differen-tial eqn for the pollutant connection is,

$$\frac{dc}{dt} = \frac{f}{V}(c_{in} - c) \ ; \ c(0) = c_0 \quad \text{—— (i)}.$$

The soln of the diff. eqn (i) is given by

$$c(t) = c_{in} - (c_{in} - c_0)\, e^{-\frac{f}{V}t}.$$

Since only fresh water is entering into the lake, $c_{in} = 0$

$$c(t) = c_0\, e^{-f/V\, t} \quad \text{—— (ii)}.$$

Given that, $c(t) = 4 \times 10^6$, $V = 28 \times 10^6 m^3$, $f = 4 \times 10^6$, $c_0 = 10^7$.

Putting these in (ii), we get —

$$4 \times 10^6 = 10^7\, e^{-\frac{4 \times 10^6}{28 \times 10^6} t}$$

$$\Rightarrow 0.4 = e^{-\frac{4 \times 10^6}{28 \times 10^6} t} = e^{-\frac{1}{7}t}.$$

$$\Rightarrow \log(0.4) = -\frac{1}{7}t \cdot \log e = -\frac{1}{7}t.$$

$$\Rightarrow t = 6.4 \text{ month}.$$