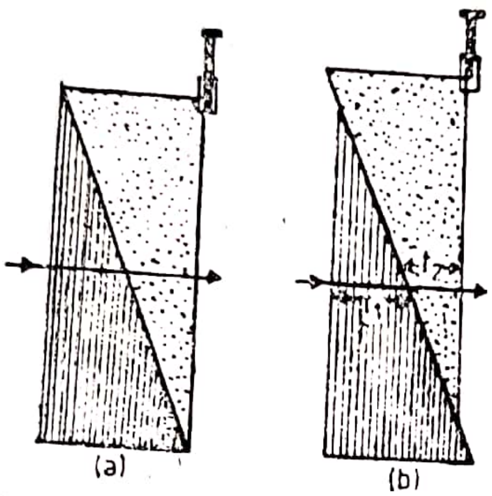
9.20. Babinet's Compensator

It is device by means of which a desired path difference can be introduced between the *O*-ray the *E*-ray for light of any wavelength.

Construction



This consists of two quartz prisms of wedge shape mounted in a holder with their hypotenuse faces adjacent and so that their optic axes are mutually perpendicular and both are perpendicular to the incident light. The faces of the wedges are cut parallel to the respective optic axis. The wedge is fixed in the holder and the other one can be moved by a micrometer screw so that its hypotenuse face slides over that of the adjacent fixed wedge.

Theory

Quartz is a positive crystal. The incident light is split up into *E*-ray and *O*-ray. The *O*-ray travels faster than *E*-ray in the wedge 1. On transmission through the interface between the wedges, the *O*-ray in wedge 1 becomes the *E*-ray in wedge 2 because the optic axis of wedge 2 is perpendicular to that of wedge 1. The speeds of two rays are interchanged at the interface.

Let t_1 and t_2 be the thickness of the two wedges traversed by transmitted light. Let μ_e and μ_o be the refractive indices of quartz for the *E*-ray and *O*-ray respectively. The path difference introduced between the two components by the first wedge is :

$$\Delta_1 = (\mu_e - \mu_o).t_1 \qquad (\because \mu_e > \mu_o \text{ for the positive crystal})$$

and that introduced by the second wedge is,

$$\Delta_2 = (\mu_o - \mu_e).\ t_2 = -(\mu_e - \mu_o)t_2.$$

Hence the total path difference

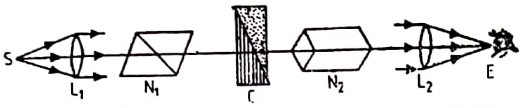
$$\Delta = \Delta_1 + \Delta_2 = (\mu_e - \mu_o)(t_1 - t_2)$$

and the corresponding phase difference is

$$\delta = \frac{2\pi}{\lambda}(\mu_e - \mu_o)(t_1 - t_2).$$

At the centre of the compensator ($t_1 = t_2$), the resultant path difference is zero so that the emergent light is plane-polarised. On either side of this point the path difference gradually increases and the emergent light is polarised in various ways.

Analysis of Elliptically Polarised Light



The compensator C is placed between crossed Nicols N_1 and N_2, oriented so that the optic axis of the wedge makes an angle of 45° with the vibration plane of the incident monochromatic light from N_1. Then a set of alternate dark and bright bands are seen. The cross-wire is placed on a dark band and the micrometer screw is moved through an angle until the next dark band is under the cross-wire. Several values of θ are determined and mean θ is evaluated. Hence the rotation θ of the micrometer eyepiece corresponds to a phase change of 2π. This serves as the calibration.

(i) Determination of Phase difference

The compensator is illuminated with white plane polarised light. The micrometer screw is adjusted to bring the central dark band under the cross-wire. The white light is replaced by the given elliptically polarised light. The central band shifts to a point where the original phase difference φ between the two component of elliptic vibration is annulled by the phase difference introduced by the compensator. The screw is rotated by θ_0 so that the central band again comes under the cross-wire. Then

$$\frac{\varphi}{2\pi}=\frac{\theta_0}{\theta};\ \therefore\ \varphi=2\pi.\frac{\theta_0}{\theta}.$$

The value of θ is obtained from calibration.

(ii) Position of Axis

The compensator is again illuminated with white plane-polarised light. The micrometer screw is adjusted to bring the central dark band under the cross-wire. The screw is then turned by $\frac{\theta}{4}$ so that the compensator introduces a phase difference of $\pi/2$. The central dark band now is not on the cross-wire.

The given elliptically polarised light is allowed to fall normally on the compensator. The compensator is rotated to bring the central dark band on the cross-wire. The axes of the incident elliptically polarised light are parallel to the optic axis of the wedges of the compensator.

(iii) Ratio of Axes

If α is the angle through which the compensator has been rotated then the ratio of axes is given by :

$$\frac{b}{a}=\tan\alpha.$$

9.21. Interference of Polarised Light

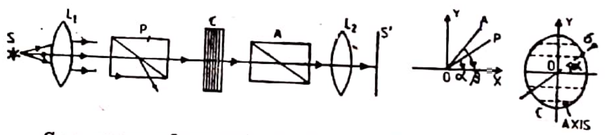
The interference of polarised light was studied by Fresnel and Arago who propounded the following laws known as **Fresnel-Arago laws.**

(i) Two rays polarised in the same plane interfere.

(ii) Two rays polarised at right angles do not interfere.

(iii) Two rays polarised at right angles interfere when brought into the same plane of polarisation provided they have been obtained from the same ray of polarised light.

Experiment :



$S\to$a source of monochromatic light of wave-length λ.

$L_1\to$a convex lens which makes light parallel.

$P\to$a Nicol prism which polarises the parallel light.

$C\to$thin plate of quartz (or calcite) cut with its faces parallel to the optic axis. This makes the light elliptically polarised.

$A\to$a Nicol prism in the crossed, position (without inserting the crystal plate C).

$L_2\to$the collimating lens.

$S'\to$a screen.

The polariser P makes the light plane-polarised. When this light passes through the crystal plate, it splits up into O-ray and E-ray. A phase difference is introduced between them as they travel with different speeds. Hence, they result in elliptically polarised light. This light falls on analysing Nicol A which transmits partly. The transmitted light consists of part of each O-component and E-

component derived from the same plane-polarised light. Analyser brings some part of these into same plane of polarisation which gives rise to interference of polarised light. Various patterns will be seen on the screen for various orientations of the polariser and analyser.

Theory

Let OX be the optic axis of the crystal C. The principal planes OP and OA of the polariser and analyser make angles α and β with OX. If the amplitude of the incident polarised light from P be a then its components along OX and perpendicular to OX will be $a \cos \alpha$ and $a \sin \alpha$ respectively. If the vibration of light incident on C be represented by

$$y = a \sin wt$$

then its components will be

$$y_e = a \cos \alpha . \sin wt \text{ (along } OX)$$

$$y_o = a \sin \alpha , \sin wt \text{ (along } OY).$$

The O-ray travels faster in quartz than the E-ray. There is a phase difference δ between them after emergence from the crystal. Hence the components incident on the analyser are

$$y'_e = a \cos \alpha . \sin wt \qquad \text{(along } OX)$$

$$y'_o = a \sin \alpha . \sin (wt + \delta) \qquad \text{(along } OY)$$

The components of vibration transmitted through the analyser A have amplitudes ($a \cos \alpha \cos \beta$) along OX and ($a \sin \alpha \sin \beta$) along OY while components ($a \cos \alpha \sin \beta$) and ($a \sin \alpha \cos \beta$) being perpendicular to OA will not be transmitted. These components are represented by

$$y''_e = a \cos \alpha \cos \beta . \sin wt$$

and $$y''_o = a \sin \alpha \sin \beta . \sin (wt + \delta).$$

The resultant vibration of the light emerging from the analyser is given by

$$y = y''_e + y''_o = a[\sin wt \, (\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos \delta) + \cos wt \, (\sin \alpha \sin \beta \sin \delta)]$$

$$= A . \sin (wt + \varphi)$$

where $A \sin \varphi = a \sin \alpha \sin \beta \sin \delta$, and

$A \cos \varphi = a (\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos \delta)$

Squaring and adding, we get for intensity,

$$I = A^2 = a^2[\cos^2 \alpha \cos^2\beta + \sin^2\alpha \sin^2\beta + \tfrac{1}{2} \sin 2\alpha \sin 2\beta - \tfrac{1}{2} \sin 2\alpha . \sin 2\beta + \tfrac{1}{2} \sin 2\alpha \sin 2\beta . \cos \delta]$$

$$= a^2[\cos^2(\beta - \alpha) - \sin 2\alpha \sin 2\beta \sin^2 \delta/2].$$

This gives the intensity of the monochromatic light transmitted through the analyser. For a **white light** source, a and δ will be different for different wavelengths and hence summing up for all wavelengths, we get

$$I = \cos^2(\beta - \alpha)\Sigma(a^2) - \sin 2\alpha \sin 2\beta \Sigma(a^2 . \sin^2 \delta/2).$$

Case I. Monochromatic Light :

When $\delta = 2n\pi$, the interference occurs and we have

$$I = a^2 \cos^2 (\beta - \alpha).$$

(i) If the Nicols P and A are crossed, then $\beta - \alpha = 90°$; $\therefore I = 0$. Then there is darkness on the screen S'.

(ii) If the Nicols P and A are parallel, then $\beta - \alpha = 0°$; $\therefore I = a^2 =$ intensity of incident light. Then there is brightness on the screen S'.

Case II. White Light

If the incident light is white, the phase difference at any point will vary from wavelength to wavelength. Hence for any position of the arrangement the intensity will be minimum for some wavelengths ($\delta = 2n\pi$) and not for all. Hence the transmitted light is coloured.

When the polariser and analyser are crossed, $\beta - \alpha = 90°$ and the resultant intensity is

$$I = \sin^2 2\alpha \Sigma(a^2 \sin^2 \delta/2).$$

The colouration is most marked.

When the analyser and polariser are parallel, $\beta - \alpha = 0°$ and the resultant intensity is

$$I = \Sigma a^2 - \sin^2 2 \alpha \, \Sigma(a^2 . \sin^2 \delta/2).$$

The colouration is the least of all.

(i) When $(\beta - \alpha) =$ constant and the plate C is rotated in its own plane, the emergent light falling on S' will be **uncoloured** for four positions of the plate corresponding to $\alpha = 0°$ and $90°$; and $\beta = 0°$ and $90°$.

(ii) When $(\beta - \alpha) = 90°$ by making $\alpha = 45°$ and $\beta = 135°$, the intensity becomes

$$I = \Sigma a^2 . \sin^2 \delta/2.$$

The colouration is maximum.

If the light from A falls on slit of a spectrometer, then the spectrum is found to be crossed by a certain number of dark lines corresponding to those wavelengths for which $\delta = 2n\pi$.