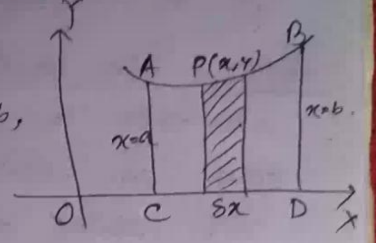
Centre of Gravity of Solid Revolution :-



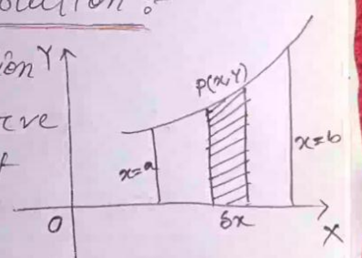
The C.G. of a solid formed by the curve $y=f(x)$ about the x-axis betn the plane $x=a$ and $x=b$, is given by,

$$\bar{x} = \frac{\int x\,dm}{\int dm} = \frac{\int_{x=a}^{b} x.\rho.\pi y^2\,dx}{\int_{x=a}^{b} \rho\pi y^2\,dx} = \frac{\int_{x=a}^{b} xy^2\,dx}{\int_{x=a}^{b} y^2\,dx}$$, where ρ is constant.

and $\bar{y}=0$.

Note:- If the curve rotated about the y axis betn the plane $y=c$ and $y=d$ then the C.G. is given by, $\bar{x}=0$. and $\bar{y} = \dfrac{\int_{y=c}^{d} yx^2\,dy}{\int_{y=c}^{d} x^2\,dy}$.

Centre of Gravity of Surface Revolution :-



The C.G. of a surface of revolution form by the revolution of the curve $y=f(x)$ about the x-axis and cut betn the planes $x=a$ and $x=b$ is given by,

$$\bar{x} = \frac{\int x\,dm}{\int dm} = \frac{\int_{x=a}^{b} x\rho\,2\pi y\,ds}{\int_{x=a}^{b} \rho\,2\pi y\,ds} = \frac{\int_{a}^{b} xy\,ds}{\int_{a}^{b} y\,ds}$$; ρ is const.

and $\bar{y}=0$.

Note:- If the surface is formed by revolving the curve about y axis betn the planes $y=c$ & $y=d$ then $\bar{x}=0$ and

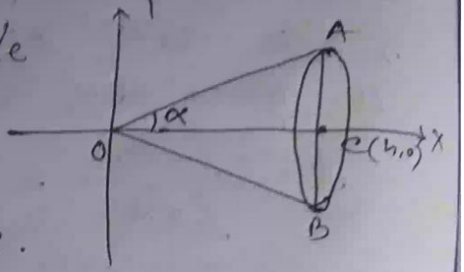
$$\bar{y} = \frac{\int y\,dm}{\int dm} = \frac{\int_{y=c}^{d} y\rho\,2\pi x\,ds}{\int_{y=c}^{d} \rho\,2\pi x\,ds} = \frac{\int_{c}^{d} xy\,ds}{\int_{c}^{d} x\,ds}.$$

1.Q. Find the c.g. of a solid and surface of a right circular of ~~height~~ hight h.

Solⁿ: For solid

We take a right angle triangle OCA in which $\angle OCA = \pi/2$.

This side OC is of length h and is along the x-axis.



If the angular area OCA revolve about x-axis, a right circular cone of hight h is generated. The x-axis will be the axis of the cone and its vertex been act at O.

Let, $\angle AOC = \alpha$.

$\therefore$ The eqⁿ of OA is $y = mx = x\tan\alpha$ —— ①.

For this cone, x varies from 0 to h.

Let, $(\bar{x}, \bar{y})$ be the reqᵈ c.g.

Then from symmetry, $\bar{y} = 0$.

and,
$$\bar{x} = \frac{\int_{x=0}^{h} xy^2\,dx}{\int_{x=0}^{h} y^2\,dx} = \frac{\int_0^h x \cdot x^2 \tan^2\alpha\,dx}{\int_0^h x^2\tan^2\alpha\,dx} = \frac{3h}{4}.$$

$\therefore$ The reqᵈ c.g. is $\left(\frac{3h}{4}, 0\right)$. //

For surface of revolution:

Clearly, from above, the eqⁿ of line OA is,

$y = mx = x\tan\alpha$ —— ①.

Diff. ①, w.r.t. x, $\frac{dy}{dx} = \tan\alpha$.

where, $\angle AOC = \alpha$.

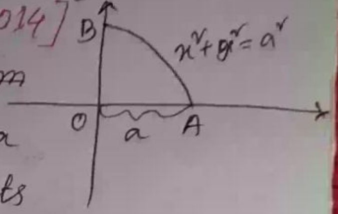
$\therefore \quad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,.dx = \sqrt{1+\tan^2\alpha}\,dx = \sec\alpha . dx.$

If, $(\bar{x}, \bar{y})$ be the reqᵈ c.g. of this surface, Then, from symmetry $\bar{y} = 0$ and

$$\bar{x} = \frac{\int_{x=0}^{h} xy\,ds}{\int_{x=0}^{h} y\,ds} = \frac{\int_0^h x \cdot x\tan\alpha \cdot \sec\alpha\,dx}{\int_0^h x\tan\alpha \cdot \sec\alpha \cdot dx} = \frac{2h}{3}.$$

Thus, the req^d c.g. is, $\left(\frac{2h}{3}, 0\right)$ //

2.Q. Find the c.g. of a solid of uniform hemisphere of radius 'a'. ['05, '06, 2014]

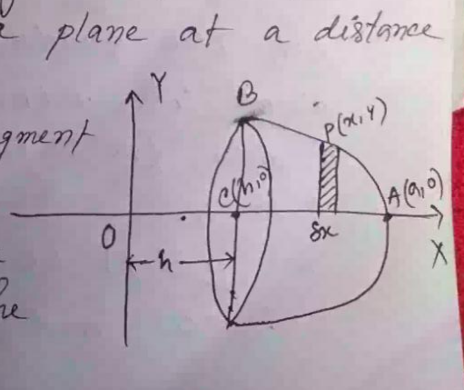


Sol^n: Let the hemesphere ~~is uniform~~ be in form by the revolution of a quadrant OAB of a circle about its bounding diameter OB which is taken along the x-axis. Since, the radius of the circle is 'a', so its eq^n is, $x^2+y^2=a^2$ —— ①.
For the hemesphere surface, x varies from 0 to a. If $(\bar{x}, \bar{y})$ be the req^d c.g. then from symmetry, $\bar{y}=0$ and

$$\bar{x} = \frac{\int_{x=0}^{a} xy^2\,dx}{\int_{x=0}^{a} y^2\,dx} = \frac{\int_0^a x(a^2-x^2)\,dx}{\int_0^a (a^2-x^2)\,dx} = \frac{3a}{8}.$$

$\therefore$ The req^d c.g. is $\left(\frac{3a}{8}, 0\right)$ //

3.Q: Find the c.g. of the segment of a sphere of radius 'a' cut off by a plane at a distance h from the centre.



Sol^n: Suppose, the spherical segment is form by revolving ~~is~~ ~~form by~~ the area ACB of a circle about x-axis. Since the radius of the sphere 'a'.

So, the eqn of the circle is $x^2+y^2=a^2$ —— ①.
Let, BC be the line $x=h$.
Then for the segment of the sphere, x varies from 'h' to 'a'.
Let, $(\bar{x},\bar{y})$ be the reqd C.G., then by symmetry, we have— $\bar{y}=0$.

And, $$\bar{x} = \frac{\int_{x=h}^{a} xy^2\,dx}{\int_{x=h}^{a} y^2\,dx} = \frac{\int_{h}^{a} x(a^2-x^2)\,dx}{\int_{h}^{a}(a^2-x^2)\,dx}.$$

$$= \left\{\frac{3}{4}\frac{(a+h)^2}{2a+h}, \cancel{0}\right\} \#$$

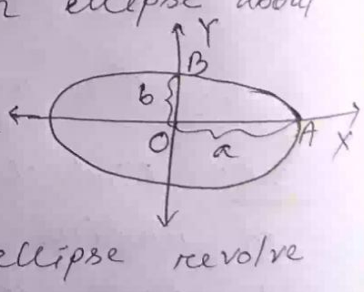
Therefore, the reqd C.G. is $\left(\frac{3}{4}\frac{(a+h)^2}{2a+h}, 0\right)$ //

Note:- In case of hemisphere, x varies from 0 to a, i.e. putting $h=0$, we have—

$$\bar{x} = \frac{3}{4}\frac{a^2}{2a} = \frac{3a}{8}.$$

∴ Reqd C.G. is, $\left(\frac{3a}{8}, 0\right)$ //

4.Q: Find the C.G. of a solid figure from by revolving a quadrant of an ellipse about its minor axis.



Soln: Let the eqn of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ —— ①.
Let, the quadrant AOB of the ellipse revolve about its minor axis OB i.e. about the y-axis,
∴ Also for this quadrant, y varies from 0 to b.
Let, $(\bar{x},\bar{y})$ be the reqd C.G. Then by symmetry, $\bar{x}=0$ and,

$$\bar{y} = \frac{\int_{y=0}^{b} y x^2 \, dy}{\int_{y=0}^{b} x^2 \, dy} = \frac{\int_0^b y \cdot \frac{1}{a^2}\left(1 - \frac{y^2}{b^2}\right) dy}{\int_0^b \frac{1}{a^2}\left(1 - \frac{y^2}{b^2}\right) dy}$$

$$= \frac{\int_0^b y\,(b^2 - y^2)\, dy}{\int_0^b (b^2 - y^2)\, dy} = \frac{3b}{8}.$$

$\therefore$ The reqd c.g. is $\left(0, \frac{3b}{8}\right)$. //