Velocity :- The rate of change of displacement with time is called velocity. It is given by $\frac{dx}{dt}$, where $x = x(t)$ gives the position of a moving particle at any time t.

Acceleration :- The rate of change of velocity with time is called acceleration. It is given by dv/dt, where $v = v(t)$ gives the velocity of a moving particle at any time t.

2. Q. Find the velocity function $v(t)$ and position function $x(t)$ of a moving particle with the given acceleration $a(t)$, initial position $x_0 = x(0)$, and initial velocity $v_0 = v(0)$, where

$a(t) = 50$, $v_0 = 10$; $x_0 = 20$.

Sol^n : We know that $a(t) = \frac{dv}{dt}$ —— (i).

Put, $a(t) = 50$ in (i), we get $\frac{dv}{dt} = 50$ —— (ii).

$$\Rightarrow dv = 50\,dt$$

Integrating, $v = 50t + c$, —— (iii)

where, c, is an integrating constant.

Put $v_0 = 10$ i.e. $v = 10$ at $t = 0$, we get —

(iii) $\Rightarrow$ $10 = 50(0) + c \Rightarrow c = 10$.

Thus, (iii) $\Rightarrow$ $v = 50t + 10$. —— (iv).

Also, $v = \frac{dx}{dt}$ —— (v).

From (iv) & (v) $\Rightarrow$ $\frac{dx}{dt} = 50t + 10$.

$$\Rightarrow dx = (50t + 10)\,dt$$

Integrating, $x = 25t^2 + 10t + c_1$ —— (vi).

Put, $x_0 = 20$, i.e. $x = 20$ at $t=0$ in (vi), we get—

$q = 20$.

From (vi), $x = 25t^2 + 10t + 20$. —— (vii).

Hence, (iv) gives the velocity function and (vii) gives position function.

3.Q. Suppose the velocity v of a motorboat coasting in water satisfies the d. equation $\frac{dv}{dt} = kv^2$. The initial speed of the motorboat is $v(0) = 10$ m/s and v is decreasing at the rate of $1\ m/s^2$ when $v = 5$ m/s. How long does it take for the velocity of the boat to decrease to 1 m/s? To ~~t~~ 1/10 m/s? When does the boat come to a stop?

Soln: Given that, $\frac{dv}{dt} = kv^2$ —— (i).

$$\Rightarrow \frac{dv}{v^2} = k\,dt.$$

Integrating, $-\frac{1}{v} = kt + c$ —— (ii).

where, c is integrating constant.

Put, $v(0) = 10$ i.e. $v = 10$ at $t = 0$ in (ii), we get—

$$c = -\frac{1}{10}.$$

From (ii), $-\frac{1}{v} = kt - \frac{1}{10}$ —— (iii).

Since, v is decreasing at the rate of $1\ m/s^2$ when $v = 5$, it means, $\frac{dv}{dt} = -1$, when $v = 5$.

Putting these values in (i), we get—

$$-1 = k(5^2) \Rightarrow k = -\frac{1}{25}.$$

Put, this value of k in (iii), we get—

$$-\frac{1}{v} = -\frac{1}{25}t - \frac{1}{10} \quad \text{——— (iv)}.$$

Now, we find t; when $v = 1$. So from (iv), we have, $t = 22{\cdot}5$.

Therefore, the motorboat will take $22{\cdot}5$ seconds for the velocity of the boat to decrease to 1 m/s.

Now, put $v = \frac{1}{10}$ in (iv), we get—

$$\frac{-1}{1/10} = \frac{-1}{25}t - \frac{1}{10} \Rightarrow t = 247{\cdot}5.$$

Therefore, the motorboat will take $247{\cdot}5$ seconds for the velocity of the boat to decreases to $1/10$ m/s.

The boat comes to stop when $v \to 0$.

It is clear from (iv), when $v \to 0$ then $t \to \infty$.

It means that, $v(t)$ approaches zero as t increases without bound.

4.Q. A stone is dropped from rest at an initial height h above the surface of the earth. Show that the speed with which it strikes the ground is $v = \sqrt{2gh}$.

Soln: When a stone is dropped from rest, at an initial height h above the surface of the earth, then $v_0 = 0$, $x_0 = 0$, $a = g$, $x = h$, $v = ?$

Now, $v = v_0 + at \Rightarrow v = 0 + gt$ ——— (i).

$\Rightarrow v = gt$

Also, $x = \frac{1}{2} at^2 + v_0 t + x_0$

$\Rightarrow h = \frac{1}{2} \cdot g \cdot t^2 + 0 \cdot t + 0.$

$\Rightarrow t^2 = \frac{2h}{g} \Rightarrow t = \sqrt{\frac{2h}{g}}.$

From ①, $v = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}.$ //

5.Q. At noon, a car starts from rest at point A and proceeds with constant acceleration along a straight road towards a point C, 35 miles away. If the constantly accelerated car arrives at C, with a velocity of 60 ml/h, at what time does it arrive at C?

Sol^n: When a car starts from rest at point A and covers a distance of 35 miles at a velocity of 60 mil/h, then $x_0 = 0$, $x = 35$, $v = 60$, $v_0 = 0$, $t = ?$

Now, $v = v_0 + at \Rightarrow 60 = 0 + at \Rightarrow at = 60$ ..

Also, $x = \frac{1}{2} at^2 + v_0 t + x_0$

$\Rightarrow 35 = \frac{1}{2} \cdot 60t + 0 \cdot t + 0$

$\Rightarrow t = 1.16$ h.

If the car starts at noon i.e. at $t = 0$ then it will take 1.16 hrs to arrive at C, means that it will arrive at 1.10 pm. //