Mathematical Models

In this chapter, we consider the problem of differential equation in co-ordinate geometry and in science and engineering. We formulate the problem mathematically thereby obtaining a differential equation, then we solve the equation.

1. Application in Coordinate Geometry :-

1.Q. In the following problems, a function $y=h(x)$ is described by some geometric property of its graph. Write a d. eqⁿ of the form $\frac{dy}{dx} = f(x,y)$ having the function h as its solution.

(i) Every straight line normal to the graph of h passes through the point $(0,1)$.

(ii) The line tangent to the graph of h at (x,y) passes through the point $(-y,x)$.

(iii) The graph of h is normal to every curve of the form $y = x^2 + k$, where k is a constant, where they meet.

Solⁿ :- (i) Slope of tangent at the point $(x,y) = \frac{dy}{dx}$

then the slope of the normal $= \frac{-1}{dy/dx}$.

The eqⁿ of straight line passing through the point $(0,1)$ and slope $\frac{-1}{dy/dx}$ is

$$(y-1) = \frac{-1}{dy/dx}(x-0)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y-1}.$$

which is the reqd eqn of the normal passes through (0, 1).

(ii) ⇒ The eqn of the tangent with slop $\frac{dy}{dx}$ and passing through the point $(-y, x)$ is,

$$y - x = \frac{dy}{dx}(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-x}{y+x}.$$

(iii) ⇒ Slope of the tangent $= m = \frac{dy}{dx}$.

slope of the normal to the curve $y = x^2 + k$ is

$$m_1 = \frac{d(x^2+k)}{dx} = 2x.$$

By condition of orthogonality, $m m_1 = -1$.

$$\Rightarrow \frac{dy}{dx} \cdot 2x = -1 \Rightarrow \frac{dy}{dx} = \frac{-1}{2x}.$$

Thus, the reqd d. eqn is, $\frac{dy}{dx} = \frac{-1}{2x}$.

2. Applications of Differential Equation in Science and Engineering :-

Newton's Second Law :- The time rate change of momentum of a body is proportional to the resultant force acting on the body and is in the direction of this resultant force.

In mathematical form,

$$\frac{d}{dt}(mv) = KF,$$ where m is the mass of body, v is velocity and F is resultant force acting upon it and K is constant of proportion-ality and m is constant, then-

$$\frac{m\,dv}{dt} = KF \Rightarrow ma = KF \Rightarrow F = \frac{1}{K} ma$$

$$\Rightarrow F = ma \text{ if } K = 1.$$