Unit-1 (CBCS – 2nd Sem)

General Sol^n :- ~~A sol^n obtained by giving particular values to the arbitrary~~ A sol^n which contains as many arbitrary constant as the order of differential equations called a general sol^n of a d. eq^n.

Particular Sol^n : A sol^n obtained by giving particular values to the arbitrary constant in the general sol^n of the d. eq^n is called a particular sol^n.

Singular Sol^n :- A sol^n which can not be obtained from general sol^n by any choice of the arbitrary constant is called a singular sol^n.

Explicit function :- The function which represent in the form $y=f(x)$ is called explicit function.
e.g. $y = x^2 + 4x - 3$, $y = ax^3 + bx$ etc.

Implicit function :- A function which can't be represent in the form $y=f(x)$ is called implicit function. e.g. $x^2+y^2=r^2$, $x^2+y^2-4=0$.

1.Q.. Solve the d. eq^n $\frac{dy}{dx} = 2x+2$.

Sol^n : Given, $\frac{dy}{dx} = 2x+2$ — (i).

$\Rightarrow dy = (2x+2)\,dx$

Integrating, $y = x^2 + 2x + c$ — (ii).

This is the explicit sol^n of (i).

2.Q. Find the explicit sol^n of $\frac{dy}{dx} = 3ax^2 + b$.

Sol^n : Given, $\frac{dy}{dx} = 3ax^2 + b$.

$\Rightarrow dy = (3ax^2+b)\,dx$

$\Rightarrow y = ax^3 + bx + c$.

which is the req^d explicit sol^n //

3.Q. Solve : $\frac{dy}{dx} = -\frac{x}{y}$.

Soln : $\frac{dy}{dx} = -\frac{x}{y} \Rightarrow x\,dx + y\,dy = 0$.

$\Rightarrow \int x\,dx + \int y\,dy = 0$.

$\Rightarrow x^2 + y^2 = c^2$.

Which is the implicit soln of given d. eqn.

4.Q. Form the d. eqn from $y = mx$, where m is arbitrary constant.

Soln : Given, $y = mx$ —①.

$\Rightarrow \frac{dy}{dx} = m$ ——— ②.

eleminating 'm' from ① and ②, we get—

$\frac{dy}{dx} = \frac{y}{x}$.

Which is the reqd d. eqn.

5.Q. Find the d. eqn of all the circles of radius 'a'.

Soln : The eqn of the circle of radius 'a' is given by,

$(x-h)^2 + (y-k)^2 = a^2$ —①.

where, h and k are the parameter.

Diff. ① w.r.t. 'x' we have—

$2(x-h) + 2(y-k)\frac{dy}{dx} = 0$. —— ②.

Again, diff. ② w.r.t. 'x', we have—

$1 + (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ —— ③.

From ② & ③, we get—

$(x-h) = -(y-k)\frac{dy}{dx}$.

and $(y-k) = -\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$.

Putting these values in (I), we get—

$$1+\left[\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2 \quad //$$

6.Q. Find the d.eqn of $x^2y = 1+cx$.

Soln: Given, $x^2y = 1+cx$ ——— (I).

Diff. (I) w.r.t. x, we get—

$$x^2\frac{dy}{dx} + 2xy = c. \quad \text{——— (II)}.$$

eliminating c from (I) & (II), we get—

$$x^2y = 1 + \left(\frac{x^2dy}{dx} + 2xy\right)x$$

$$\Rightarrow x^3\frac{dy}{dx} + x^2y + 1 = 0.$$

Which is the reqd soln. //

Home Work

1.Q. Find the d.eqn of $y = ae^{2x} + be^{-3x} + ce^x$, where a, b, c are arbitrary constants.

2.Q. Find the d.eqn of the family of curves $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants.

3.Q. Find the d.eqn of the family of curves $y = Ae^x + \frac{B}{e^x}$, for different values of A and B.

4.Q. Show that, $Ax^2 + By^2 = 1$ is the solution of

$$x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] - y\frac{dy}{dx} = 0.$$