Unit - 2 (4th sem)

Centre of Gravity (C.G.) :→

The centre of gravity of a body or a system of body connected togethers is that point through which the line of action of the body always passes.

Note: 1. The C.G. of the body doesnot necess--arily lie in the body itself.

2. If a body is symmetrical about a line, the C.G. of the body always lies on the line of symmetrical.

3. The centre of mass of a body practically coinside with its centre of gravity. Sometimes the word centroid or centre of inertia are use, in place of C.G.

Determination of C.G. by Integration :-

If a number of particle of masses $m_1, m_2, \ldots$ be placed at the point whose co-ordinates are $(x_1, y_1), (x_2, y_2), \ldots$ referred to two rectangular axes and $(\bar{x}, \bar{y})$ be the coordinate of C.G. of the body consisting by the particles. Then the C.G. of the body is given by,

$$\bar{x} = \frac{\sum m_i x_i}{\sum m} \quad \text{and} \quad \bar{y} = \frac{\sum m_i y_i}{\sum m}.$$

In case of continuous distribution, ~~of~~ the Σ can be replaced by definite integrals. Then the C.G. of the body is given by,

$$\bar{x} = \frac{\int x\, dm}{\int dm} \quad \text{and} \quad \bar{y} = \frac{\int y\, dm}{\int dm}.$$

where, (x, y) be the c.G. of an elementory mass dm of the given matter. The limit being chosen so as to include all the portion consider.

In case of an arc, the c.G. is given by,

$$\bar{x} = \frac{\int \rho x \, ds}{\int \rho \, ds} \quad \text{and} \quad \bar{y} = \frac{\int \rho y \, ds}{\int \rho \, ds}.$$

where, (x, y) is any point on the elementary arc ds. The limit of integration being consider from one in two the other of the arc consider. ρ is the density any point on the arc.

Note: In the application of the above integrals, the following results are follows:

(i) If, $y = f(x)$ then, $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, . \, dx$

(ii) If, $x = f(y)$ then, $ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, . \, dy$

(iii) If, $x = \phi(t)$, $y = \psi(t)$ then $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, . \, dt$.

(iv) If, $f(r, \theta) = 0$ then $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, . \, d\theta$

or $ds = \sqrt{1 + r^2 \left(\frac{d\theta}{dr}\right)^2} \, . \, dr$.

where, $x = r\cos\theta$, $y = r\sin\theta$.

C.G. of a plane area :- Let, $(\bar{x}, \bar{y})$ be the co-ordinate of the c.G. of an area A. Then,

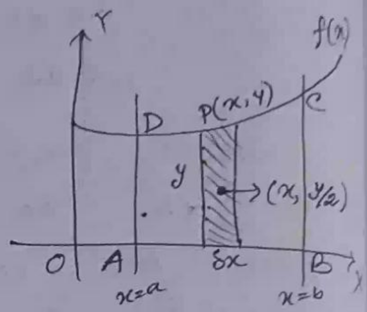
$$\bar{x} = \frac{\int x \, dm}{\int dm} = \frac{\int x \rho \, dA}{\int \rho \, dA} \quad \text{and}$$

$$\bar{y} = \frac{\int y \, dm}{\int dm} = \frac{\int y \rho \, dA}{\int \rho \, dA}.$$

Where, ρ is the density of the area and dA be an elementary mass.

Here, two cases aries;

<u>case I</u>: To find the c.G. of a plane area bounded by the curve $y = f(x)$, the x axis and the ordinates $x = a$ and $x = b$.



Suppose, we have to find the c.G. of the area ABCD for this, we consider an elementary strip at $P(x, y)$ of thickness δx.

Then its c.G. will lie at its midpoint $(x, y/2)$. Therefore the c.G. of the area ABCD will be given by,

$$\bar{x} = \frac{\int \rho x \, dA}{\int \rho \, dA} = \frac{\int_{x=a}^{b} x \cdot y \, dx}{\int_{x=a}^{b} y \, dx} \quad \text{and}$$

$$\bar{y} = \frac{\int \rho \frac{y}{2} \, dA}{\int \rho \, dA} = \frac{\int_{x=a}^{b} \frac{y}{2} \cdot y \, dx}{\int_{x=a}^{b} y \, dx} = \frac{1}{2} \frac{\int_{x=a}^{b} y^2 \, dx}{\int_{x=a}^{b} y \, dx}.$$

Where, ρ is considered as const. and dA = elementary area $= y \cdot \delta x$.

<u>case II</u>: The c.G. of a plane area bounded by the curve $x = f(y)$, the y axis and the abscissae $y = c$ and $y = d$ are given by,

$$\bar{x} = \frac{1}{2} \frac{\int_{y=c}^{d} x^2 \, dy}{\int_{y=c}^{d} x \, dy} \quad \text{and} \quad \bar{y} = \frac{\int_{y=c}^{d} xy \, dy}{\int_{y=c}^{d} x \, dy}.$$