Biot Savart Law

The **Biot Savart Law** is used to determine the magnetic field intensity H near a current-carrying conductor or we can say, it gives the relation between magnetic field intensity generated by its source current element. The law was stated in the year 1820 by Jean Baptisle**Biot** and Felix **Savart.** The direction of the magnetic field follows the right hand rule for the straight wire. **Biot Savart** law is also known as Laplace’s law or Ampere’s law.

**Consider a wire carrying an electric current I and also consider an infinitely small length of a wire dl at a distance x from point A.**



**Biot Savart** Law states that

The magnetic intensity dH at a point A due to current I flowing through a small element dl is

1. Directly proportional to current (I)
2. Directly proportional to the length of the element (dl)
3. Directly proportional to the sine of angle θ between the direction of current and the line joining the element dl from point A.
4. Inversely proportional to the square of the distance (x) of point A from the element dl.



where k is constant and depends on the magnetic properties of the medium.


µ0 = absolute permeability of air or vacuum and its value is 4 x 10-7 Wb/A-m
µr= relative permeability of the medium.

## Biot Savart Law

We know that electric current or moving charges are source of magnetic field

A Small current carrying conductor of length **dl** (length element ) carrying current I is a elementary source of magnetic field .The force on another similar conductor can be expressed conveniently in terms of magnetic field **dB** due to the first

The dependence of magnetic field **dB** on current I ,on size and orientation of the length element **dl** and on distance **r** was first guessed by Biot and savart

The magnitude of the magnetic field **dB** at a distance **r** from a current element **dl** carrying current I is found to be proportional to I ,to the length dl and inversely proportional to the square of the distance |**r**|

The direction of the magnetic Field is perpendicular to the line element **dl** as well as radius **r**

Mathematically, Field **dB** is written as


Here (μ0/4π) is the proportionality constant such that
μ0/4π=10-7 Tesla Meter/Ampere(Tm/A)

Figure below illustrates the relation between magnetic field and current element if in figure,



Consider that line element **dl** and radius vector **r** connecting line element mid point to the field point P at which field is to be found are in the plane of the paper

From equation (1) ,we expect magnetic field to be perpendicular to both **dl** and **r**.Thus direction of **dB** is the direction of advance of right hand screw whose axis is perpendicular to the plane formed by **dl** and **r** and which is rotated from **dl** to **r** ( right hand screw rule of vector product)

Thus in figure ,**dB** at point P is perpendicular directed downwards represented by the symbol (x) and point Q field is directed in upward direction represented by the symbol (•)

The magnitude of magnetic field is

where θ is the angle between the line element dl and radius vector **r**

The resultant field at point P due to whole conductor can be found by integrating equation (1) over the length of the conductor i.e.
**B**=∫d**B**

**Relation between permeability (μ0 and permittivity (ε0) of the free space**

We know that
μ0/4π=10-7 N/A2 ----(a)
and 1/4πε0=9\*109 N-m2/C2 ----(b)
Dividing equation (a) by (b) we get
μ0ε0 =1/(9\*1016) (C/Am)2
we know that 1C=1A-s

So μ0ε0 =1/(3\*108 m/s)2
And 3\*108 m/s is the speed of the light in free space So μ0ε0 =1/c2 or c=1/√(μ0ε0)

## Applications of Biot Savart law

Biot Savart law has much application. In this section we will now apply Biot-Savart law as studied in previous section to calculate field B in some important cases.

## (i) Magnetic Field due to steady current in an infinitely long straight wire

Consider a straight infinitely long wire carrying a steady current I

We want to calculate magnetic field at a point P at a distance R from the wire as shown below in figure


From Biot,-Savart law ,magnetic field d**B** due to small current element of the wire at point O at a distance |**r**|=r from point P is



since current element Id**l** and vector **r** makes an angle θ with each other ,the magnitude of the product d**l**X**r** is dlrsinθ and is directed perpendicular to both dl and r vector as shown in the figure


Since from our choice of co-ordinate, we found out that field B lies along z-axis therefore we can write



where **k** is the unit vector along z-axis

we will now express sinθ and r in terms of R which is fixed distance for any point in space and l which describes the position of current element on the infinitely long wire .From figure 1 we have



and r=(R2 +l2)1/2
Putting these values in the equation (5) we find



To find the field due to entire straight wire carrying wire ,we would have to integrate equation (6) **B**=∫d**B**



To evaluate the integral on the RHS substitute
l=RtanΦ and dl=Rsec2Φ dΦ
Therefore



From equation (7) ,we noticed that
i) Magnetic field is proportional to the current I
ii) It is inversely proportional to the distance R
iii)Magnetic field is in the direction perpendicular to the straight wire and vector **AP**=**R**

The magnetic line of force near a linear current carrying wire is concentric circles around the conductor in a plane perpendicular to the wire

Hence the direction of field Bat point P at a distance R from wire, will be along the tangent drawn on a circle of radius R around the conductor as shown below in figure



**Direction of B**

Direction of B can be found by right hand thumb rule i.e. grasp the wire with right hand ,the thumb pointing in direction of current ,the finger will curl around the wire in the direction of **B**

The magnetic field lines are circular closed curve around the wire

## (ii) Force between two long and parallel current carrying

## (ii) Force between two long and parallel current carrying conductor

It is experimentally established fact that two current carrying conductors attract each other when the current is in same direction and repel each other when the current are in opposite direction

Figure below shows two long parallel wires separated by distance d and carrying currents I1 and I2



Consider fig 5(a) wire A will produce a field B1 at all near by points .The magnitude of B1 due to current I1 at a distance d i.e. on wire b is
B1=μ0I1/2πd                   ----(8)

According to the right hand rule the direction of **B1** is in downward as shown in figure (5a)

Consider length l of wire B and the force experienced by it will be (I2**l**X**B**) whose magnitude is



Direction of F2 can be determined using vector rule .F2 Lies in the plane of the wires and points to the left

From figure (5) we see that direction of force is towards A if I2 is in same direction as I1 fig( 5a) and is away from A if I2 is flowing opposite to I1 (fig 5b)

Force per unit length of wire B is



Similarly force per unit length of A due to current in B is


and is directed opposite to the force on B due to A. Thus the force on either conductor is proportional to the product of the current

We can now make a conclusion that the conductors attract each other if the currents are in the same direction and repel each other if currents are in opposite direction

## (iii) Magnetic Field along axis of a circular current carrying coil

Let there be a circular coil of radius R and carrying current I. Let P be any point on the axis of a coil at a distance x from the center and which we have to find the field

To calculate the field consider a current element Id**l** at the top of the coil pointing perpendicular towards the reader

Current element Id**l** and **r** is the vector joining current element and point P as shown below in the figure



From Biot Savart law, the magnitude of the magnetic field due to this current element at P is


where Φ is the angle between the length element dl and r

Since Id**l** and **r** are perpendicular to each other so Φ=90.Therefore



Resolving dB into two components we have dBsinθ along the axis of the loop and another one is dBcosθ at right angles to the x-axis

Since coil is symmetrical about x-axis the contribution d**B** due to the element on opposite side ( along -y axis ) will be equal in magnitude but opposite in direction and cancel out. Thus we only have dBsinθ component

The resultant B for the complete loop is given by,
B=∫dB


Now from figure 6
sinθ=R/r =R/√(R2 + x2) So
eq



If the coil has N number of turns then



**Direction of B**
Direction of magnetic field at a point on the axis of circular coil is along the axis and its orientation can be obtained by using right hand thumb rule .If the fingers are curled along the current, the stretched thumb will point towards the magnetic field

Magnetic field will be out of the page for anti-clockwise current and into the page for clockwise direction

**Field at center of the coil**
At the center of the coil x=0
so


**Field at point far away from the center x>>>R**
In this case R in the denominator can be neglected hence



For coil having N number of turns



If the area of the coil is πR2 then



m=NIA represents the magnetic moment of the current coil. Thus from equation (17) we have


## (iv) Magnetic Field at the center of a current carrying arc

* Consider an arc of radius R carrying current I as shown below in the figure



According to the Biot Savart law the magnetic field at any point P is given by


Here dl=RdΦ
So



If l is the length of the arc then
l=RΦ so that



Equation 19 and 20 gives us magnetic field only at the center of curvature of a circular arc of current

For semi-circular loop put Φ=π in equation 19 and for full circle Φ=2π in equation 19 and calculate to find the result

If the circular current loop lies on the plane of the paper then magnetic field will be out of the page for anticlockwise current and into the page for clockwise current as shown below in the figure



## Ampere's circuital law

Ampere's circuital law in magnetism is analogous to gauss's law in electrostatics

This law is also used to calculate the magnetic field due to any given current distribution

This law states that
" The line integral of resultant magnetic field along a closed plane curve is equal to μ0 time the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant"



 Thus where μ0 is the permeability of free space and Ienc is the net current enclosed by the loop as shown below in the figure


The circular sign in equation (21) means that scalar product **B**.**dl** is to be integrated around the closed loop known as Amperian loop whose beginning and end point are same

Anticlockwise direction of integration as chosen in figure 9 is an arbitrary one we can also use clockwise direction of integration for our calculation depending on our convenience

To apply the ampere's law we divide the loop into infinitesimal segments **dl** and for each segment, we then calculate the scalar product of **B** and **dl**

**B** in general varies from point to point so we must use **B** at each location of **dl**

Amperian Loop is usually an imaginary loop or curve, which is constructed to permit the application of ampere's law to a specific situation

**Proof Of Ampere's Law**

Consider a long straight conductor carrying current I perpendicular to the page in upward direction as shown below in the figure



From Biot Savart law, the magnetic field at any point P which is at a distance R from the conductor is given by



Direction of magnetic Field at point P is along the tangent to the circle of radius R withTh conductor at the center of the circle

For every point on the circle magnetic field has same magnitude as given by

And field is tangent to the circle at each point

The line integral of B around the circle is


since ∫dl=2πR ie, circumference of the circle so,


This is the same result as stated by Ampere law

This ampere's law is true for any assembly of currents and for any closed curve though we have proved the result using a circular Amperian loop

If the wire lies outside the Amperian loop, the line integral of the field of that wire will be zero


but does not necessarily mean that **B**=0 everywhere along the path ,but only that no current is linked by the path

while choosing the path for integration ,we must keep in mind that point at which field is to be determined must lie on the path and the path must have enough symmetry so that the integral can be evaluated

## Magnetic field of a solenoid

A solenoid is a long wire wound in a close-packed helix carrying a current I and the length of the solenoid is much greater than its diameter

Figure below shows a section of a stretched out solenoid in xy and yz plane



The solenoid magnetic field is the vector sum of the field produced by the individual turns that make up the solenoid

Magnetic field B is nearly uniform and parallel to the axis of the solenoid at interior points near its center and external field near the center is very small

Consider a dashed closed path abcd as shown in figure .Let l be the length of side ab of the loop which is parallel to the is of the solenoid

Let us also consider that sides bc and da of the loop are very-very long so that side cd is very much far away from the solenoid and magnetic field at this side is negligibly small and for simplicity we consider its equal to 0

At side a magnetic field **B** is approximately parallel and constant. So for this side
∫B.dl=Bl

Magnetic field B is perpendicular to sides bc and da ,hence these portions of the loop does not make any contributions to the line integral as **B**.**dl**=0 for the side bc and da

Side cd lies at external points solenoid where **B**.**dl**=0 as B=0 or negligibly small outside the solenoid

Hence sum around the entire closed path reduces to Bl

If N are number of turns per unit length in a solenoid then number of turns in length l is nl.The total current through the rectangle abcd is NIl and from ampere 's law

Bl=μ0NlI
or B=μ0NI                    (22)

we have obtained this relation for infinitely long solenoids considering the field at external points of the solenoid equal to zero.

However for real solenoids external field is relatively weak rather then equal to zero

Thus for actual solenoids relation 22 holds for internal points near the center of the solenoid

Field at internal points of the solenoid does not depend on length and diameter of the solenoid and is uniform over the cross-section of a solenoid