Norms of a matrix

Defination:- Norms of a matrix is defined as the square root of the sum of the absolute squares of ~~its ele~~ each elements and its denoted by $\|N\|$

Formula :- $\|N\| = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2}$

Properties of Norms

(i) For a matrix A, $\|A\| \geq 0$

(ii) For a matrix A and a scalar K, $\|KA\| = \|K\| \, \|A\|$.

(iii) For a matrix A and B of same order, $\|A+B\| \leq \|A\| + \|B\|$

(iv) For a matrix A and B which can be multiplied AB,

$$\|AB\| \leq \|A\| \, \|B\|$$

Scanned with CamScanner

For ex:- (i) $A = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}_{2\times 2}$, find norms of A

Sol^n

$$\|A\| = \sqrt{\sum a_{ij}^2}$$

$$= \sqrt{2^2 + 3^2 + 5^2 + 1^2}$$

$$= \sqrt{4+9+25+1}$$

$$= \sqrt{39} = 6.24$$

Trace of a Matrix

Defination was given (check pic.)

~~Pro~~ Law of trace of a matrix

(i) $Tr(A+B) = Tr(A) + Tr(B)$

(ii) $Tr(A-B) = Tr(A) - Tr(B)$

(iii) $Tr(KA) = K\,Tr(A)$

(iv) $Tr(AB) = Tr(BA)$

Scanned with CamScanner

(i) $Tr(A+B) = Tr(A) + Tr(B)$ Prove it.

Sol^n

$L.H.S = Tr(A+B)$

Let, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}_{2\times 2}$

$$Tr(A+B) = \begin{bmatrix} 1+3 & 2+4 \\ 3+5 & 4+6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix}_{2\times 2}$$

$\therefore Tr(A+B) = 4 + 10$

$= 14$

$RHS = Tr(A) + Tr(B)$

~~Tr(A) = 1+4 = 5~~ $Tr(A) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$

~~Tr(B) =~~

$= 1+4$

$= 5$

$Tr(B) = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}_{2\times 2}$

$= 3+6$

$= 9$

$\therefore Tr(A) + Tr(B) = 5 + 9 = 14$

$\therefore Tr(A+B) = Tr(A) + Tr(B)$ proved.

(*) Try yourself the other three law