**Impedance, Admittance, and Hybrid or h-parameters BSc General 6th semester**

**FOUR TERMINAL NETWORKS**

**A four terminal network, also called two terminal pair network, is treated as a black box with two input terminals and two output terminals. The black box may contain linear network (either active or passive) such as transformer, filter, transistor and amplifier. A four terminal network is also called two port network if for all possible external connections at these terminals, the current entering the network by terminal 1 is equal to that leaving the network by terminal** $\acute{1}^{'}$ **and the current entering the network by terminal 2 is equal to that leaving the network by terminal** $\acute{2}^{'}$ **. The port at the left is referred to as input port and the port at the right is referred to as output port.**

**The equivalent two port parameters are calculated from current and voltage measurements made at the input and output terminals. Let input voltage and current at terminal 1**$\acute{1}^{'}$ **be** $v\_{1}$ **and** $i\_{1}$ **and output voltage and current be** $v\_{2}$ **and** $i\_{2}$**. Any pair of these variables may be arbitrarily chosen as independent, giving two equations. The solution of these equations gives two other dependent variables.**

**2**

$$i\_{1}$$

$$i\_{2}$$

$$v\_{1}$$

$$v\_{2}$$

**1**  +

$1^{'}$

$$2^{'}$$

**NETWORK**

Impedance Parameter or z-parameter:

Making the choice of input current $i\_{1}$ and output current $i\_{2}$as independent variables implies the general relations- $v\_{1}$**=f(**$i\_{1},i\_{2})$and $v\_{2}$**=f(**$i\_{1},i\_{2})$

These may be written as two set of equations :

d$v\_{1}$**= (**$∂v\_{1}/∂i\_{1})di\_{1 }+(∂v\_{1}/∂i\_{2})di\_{2}$

d$v\_{2}$**= (**$∂v\_{2}/∂i\_{1})di\_{1 }+(∂v\_{2}/∂i\_{2})di\_{2}$

For sufficiently small a-c signal these equations can be written as:

$V\_{1}=z\_{11}I\_{1}+z\_{12}I\_{2}$ **----(1)**

$V\_{2}=z\_{21}I\_{1}+z\_{22}I\_{2}$ **----(2)**

$\left[\begin{matrix}V\_{1}\\V\_{2}\end{matrix}\right]=\left[\begin{matrix}z\_{11}&z\_{12}\\z\_{21}&z\_{22}\end{matrix}\right]\left[\begin{matrix}I\_{1}\\I\_{2}\end{matrix}\right]$ **---(3)**

Here z-parameters have been defined as the slope of the **v-i** relations and are known as open circuit impedance parameters of the network and are useful in general network analysis but not for transistor studies.

Using $I\_{1}$**=0** and $I\_{2}$**=0** in the above equations(1 and 2) we have-

$z\_{11}= ∂v\_{1}/∂i\_{1}$**=** Open circuit input impedance ($I\_{2}$**=0)**

$z\_{12}= ∂v\_{1}/∂i\_{2}$**=** Open circuit reverse transfer impedance ($I\_{1}$**=0)**

$z\_{21}= ∂v\_{2}/∂i\_{1}$**=** Open circuit forward transfer impedance ($I\_{2}$**=0)**

$z\_{22}= ∂v\_{2}/∂i\_{2}$**=** Open circuit output impedance ($I\_{1}$**=0)**

Equation (3) may be written as **V=ZI,** where the matrix **Z=**$\left[\begin{matrix}z\_{11}&z\_{12}\\z\_{21}&z\_{22}\end{matrix}\right]$is called the open circuit impedance matrix of two port network. Z-parameters may be determined experimentally from phasor current and phasor voltage measurements made during open circuit tests.

**Admittance Parameters**.

Making the choice of input voltage $v\_{1}$ and output voltage $v\_{2}$as independent variables implies the general relations- $i\_{1}$**=f(**$v\_{1},v\_{2})$and $i\_{2}$**=f(**$v\_{1},v\_{2})$

These may be written as two set of equations :

d$i\_{1}$**= (**$∂i\_{1}/∂v\_{1})dv\_{1 }+(∂i\_{1}/∂v\_{2})dv\_{2}$

d$i\_{2}$**= (**$∂i\_{2}/∂v\_{1})dv\_{1 }+(∂i\_{2}/∂v\_{2})dv\_{2}$

For sufficiently small a-c signals where the partial derivatives are constant these equations can be written as:

$I\_{1}=y\_{11}V\_{1}+y\_{12}V\_{2}$ **----(4)**

$I\_{2}=y\_{21}V\_{1}+y\_{22}V\_{2}$ **----(5)**

$\left[\begin{matrix}I\_{1}\\I\_{2}\end{matrix}\right]=\left[\begin{matrix}y\_{11}&y\_{12}\\y\_{21}&y\_{22}\end{matrix}\right]\left[\begin{matrix}V\_{1}\\V\_{2}\end{matrix}\right]$ **---(6)**

Here y-parameters are known as short circuit admittance parameters. Using $V\_{1}$**=0** and $V\_{2}$**=0** in the above equations(4 and 5) we have-

$y\_{11}= ∂i\_{1}/∂v\_{1}$**=** Short circuit input admittance ($V\_{2}$**=0)**

$y\_{12}= ∂i\_{1}/∂v\_{2}$**=** Short circuit reverse transfer admittance ($V\_{1}$**=0)**

$y\_{21}= ∂i\_{2}/∂v\_{1}$**=** Short circuit forward transfer admittance ($V\_{2}$**=0)**

$y\_{22}= ∂i\_{2}/∂v\_{2}$**=** Short circuit output admittance ($V\_{1}$**=0)**

Equation (6) may be written as **I=YV,** where the matrix **Y=**$\left[\begin{matrix}y\_{11}&y\_{12}\\y\_{21}&y\_{22}\end{matrix}\right]$is called the short circuit admittance matrix of two port network. Y-parameters may be determined experimentally from phasor current and phasor voltage measurements made during short circuit tests.

It is easy to prove from equation (3) and (6) that $Z=Y^{-1}$or $Y=Z^{-1}$

**Hybrid Parameters-** The third choice of input current $i\_{1}$ and output voltage $v\_{2}$as independent variables leads to a set of network parameters of considerable value in transistor circuit analysis.

It gives $v\_{1}$**=f(**$i\_{1},v\_{2})$and $i\_{2}$**=f(**$i\_{1},v\_{2})$

d$v\_{1}$**= (**$∂v\_{1}/∂i\_{1})di\_{1 }+(∂v\_{1}/∂v\_{2})dv\_{2}$

d$i\_{2}$**= (**$∂i\_{2}/∂i\_{1})di\_{1 }+(∂i\_{2}/∂v\_{2})dv\_{2}$

For sufficiently small a-c signals where the partial derivatives are constant these equations can be written as:

$V\_{1}=h\_{11}I\_{1}+h\_{12}V\_{2}$ **----(7)**

$I\_{2}=h\_{21}I\_{1}+h\_{22}V\_{2}$ **-----(8)**

$\left[\begin{matrix}V\_{1}\\I\_{2}\end{matrix}\right]=\left[\begin{matrix}h\_{11}&h\_{12}\\h\_{21}&h\_{22}\end{matrix}\right]\left[\begin{matrix}I\_{1}\\V\_{2}\end{matrix}\right]$ **---(9)**

Here h-parameters are known as hybrid parameters, as they are mixed parameters compared to pure impedance and admittance parameters. These are defined under either open or short circuit conditions. We have $V\_{2}=0$for a short circuit and $I\_{1}=0$ for an open circuit. Thus we have two sets of h-parameters, as defined below:

$h\_{11}= ∂v\_{1}/∂i\_{1}$**=** Short circuit input impedance ($V\_{2}=0)$

$h\_{12}= ∂v\_{1}/∂v\_{2}$**=** Open circuit reverse voltage gain ($I\_{1}=0)$

$h\_{21}$**=**$ ∂i\_{2}/∂i\_{1}$**=** Short circuit forward current gain ($V\_{2}=0)$

$h\_{22}= ∂i\_{2}/∂v\_{2}$ **=** Open circuit output admittance ($I\_{1}=0)$

Here one parameter is an impedance, one an admittance and two are dimensionless ratios. The condition for the two port to be reciprocal is that $h\_{21}$**= -** $h\_{12} $

In equation (9) may define the matrix $\left[\begin{matrix}h\_{11}&h\_{12}\\h\_{21}&h\_{22}\end{matrix}\right]$**=H**

The matrix **H** is called a hybrid matrix. The hybrid matrices describe a two port when the voltage of one port and the current of the other are taken as the independent variables. The h-parameters may be determined experimentally from phasor current and phasor voltage measurements made during output port short-circuited and input port open-circuited. These three sets of parameters have been found useful in electronic circuit analysis. They can be measured under the specified conditions or evaluated from the characteristics. If one set of parameters are given, other two can be calculated from equations (3), (6) and (9).

To demonstrate the Hybrid transistor model an ac equivalent circuit is shown below:

$$i\_{1}$$

$$h\_{11}$$

**1**

$$1^{'}$$

**2**

$$2^{'}$$

$$v\_{1}$$

$$i\_{2}$$

$$h\_{22}$$

$$h\_{21}i\_{1}$$

$$h\_{12}v\_{2}$$

$$v\_{2}$$

The h-parameter model will be applied to a single common emitter (CE) stage with the emitter resistor (RE) bypassed. The model will be used to build equations for voltage gain($A\_{v})$, current gain ($A\_{i})$, input ( $Z\_{i})$and output impedance $(Z\_{o})$. The circuit is shown below:



The hybrid model has four h-parameters. The "h" stands for hybrid because the parameters are a mix of impedance, admittance and dimensionless units. In common emitter the parameters are:

|  |  |
| --- | --- |
| **hie** | **input impedance (Ω)** |
| **hre** | **reverse voltage ratio (dimensionless)** |
| **hfe** | **forward current transfer ratio (dimensionless)** |
| **hoe** | **output admittance (Siemen)** |

Note that lower case suffixes indicate small signal values and the last suffix indicates the mode so hie is input impedance in common emitter, hfb would be forward current transfer ration in common base mode, etc. The hybrid model for the BJT in common emitter mode is shown below:



The hybrid model is suitable for small signals at mid band and describes the action of the transistor. Two equations can be derived from the diagram, one for input voltage vbe and one for the output ic:

**vbe = hie ib + hre vce
   ic = hfe ib + hoe vce**

The small signal parameter hreVce is often too small to be considered so the input resistance is just hie. Often the output resistance hoe is often large compared wi the the collector resistor RC and its effects can be ignored. The h-parameter equivalent model is now simplified and drawn below:



The input impedance is the parallel combination of bias resistors RB1 and RB2. As the power supply is considered short circuit at small signal levels then RB1 and RB2 are in parallel. RBB will represent the parallel combination:

|  |  |
| --- | --- |
| RBB = RB1 || RB2 = | RB1 RB2 |
| RB1 + RB2 |

As RBB is in parallel with hie then:

Zi = RBB || hie

OutpImpedance Zo
As hfeIb is an ideal current generator with infinite output impedance, then output impedance looking into the circuit is: Zo = RC

Note the − sign in the equation, this indicates phase inversion of the output waveform.
Vo = -Io $R\_{C}$ = -hfe Ib $R\_{C}$as Ib = Vi / hie then:

|  |  |  |
| --- | --- | --- |
| = -hfe | Vi | $$R\_{C}$$ |
| hie |
| = | -hfe | $R\_{C}$ Vi |
| hie |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Av = | Vo |  =  | -hfe  | $$R\_{C}$$ |
| Vi |    hie |

The current gain is the ratio Io / Ii. At the input the current is split between the parallel branch RBB and hie. So looking at the equivalent h-parameter model again (shown below):



The current divider rule can be used for Ib:

|  |  |
| --- | --- |
| Ib = | RBB Ii |
| RBB + hie |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Ib |  =  | RBB |  |
| Ii | RBB + hie |

At the output side, Io = hfe Ib re-arranging Io / Ib = hfe

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai = | Io |  = | Io Ib |  = hfe | RBB |  |
| Ii | Ib Ii | RBB + hie |

If RBB >> hie then,

|  |  |  |
| --- | --- | --- |
| Ai ≈ |   RBB hfe |  = hfe |
|    RBB |

Summary
The hybrid model is limited to a particular set of operating conditions for accuracy. If the device is operated at a different collector current, temperature or Vce level from the manufacturers datasheet then the h parameters will have to be measured for these new conditions. The hybrid model has parameters for output impedance and reverse voltage ratio which can be important in some circuits.

The table below lists the four h-parameters for the BJT in common base and common collector (emitter follower) mode-

|  |  |  |  |
| --- | --- | --- | --- |
| CommonBase | Common Emitter | CommonCollector |      Definitions |
| hib | hie | hic | Input Impedance  withOutput Short Circuit |
| hrb | hre | hrc | Reverse Voltage RatioInput Open Circuit |
| hfb | hfe | hfc | Forward Current GainOutput Short Circuit |
| hob | hoe | hoc | Output AdmittanceInput Open Circuit |

H-parameters are not constant and vary with temperature, collector current and collector emitter voltage. For this reason when designing a circuit the hybrid parameters should be measured under the same conditions as the actual circuit.