17.10. DIFFRACTION PATTERN DUE TO A STRAIGHT EDGE

Let S be narrow slit illuminated by a source of monochromatic light of wavelength, λ. The length of the slit is perpendicular to the plane of the paper. AD is the straight edge and the length of the edge is parallel to the length of slit (Fig.17.15). XY is the incident cylindrical wavefront. P is a

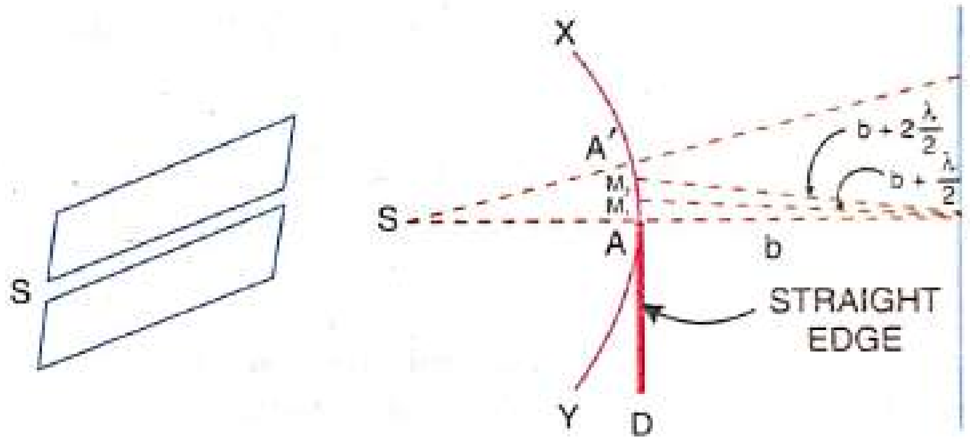

Fig. 17.15

point on the screen and SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. Below the point P is the geometrical shadow and above P is the illuminated portion. Let the distance AP be b. With reference to the point P, the wave front can be divided into a number of half period strips, as shown in Fig. 17.16. XY is the wave front, A is the pole of the wave front and AM_1, M_1M_2, M_2M_3 etc measure of the thickness of the 1st, 2nd, 3rd etc half period strips. With the increase in the order of the strip, the area of the strip decreases (Fig. 17.16).

In Fig.17.15, AP = b,

$$PM_1 = b + \frac{\lambda}{2} \quad \text{and} \quad PM_2 = b + \frac{2\lambda}{2} \text{ etc.}$$

Let P′ be a point on the screen in the illuminated portion (Fig. 17.17). To calculate the resultant effect at P′ due to the wave front XY, let us join S to P′. This line

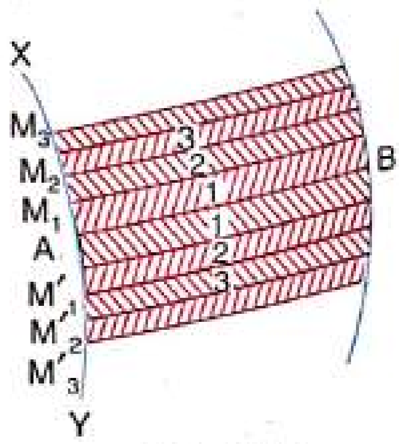

Fig. 17.16

meets the wave front at B. B is the pole of the wave front with reference to the point P′ and the intensity at P′ will depend mainly on the number of half period strips enclosed between the points A and B. The effect at P′ due to the wave front above B is same at all points on the screen whereas it is different at different points due to the wave front between B and A. The point P′ will be of maximum intensity, if the number of half period strips enclosed between B and A is odd and the intensity at P′ will be minimum if the number of half period strips enclosed between B and A is even.

17.10.1. POSITIONS OF MAXIMUM AND MINIMUM INTENSITY

Let the distance between the slit and the straight edge be a and the distance between the straight edge and the screen be b (Fig. 17.17). Let PP′ be x.

The path difference, $\delta = AP' - BP'$

$$= (b^2 + x^2)^{1/2} - [SP' - SB]$$

$$= (b^2 + x^2)^{1/2} - \left(\sqrt{(a+b)^2 + x^2} - a\right)$$

$$= b\left[1 + \frac{x^2}{2b^2}\right] - (a+b)\left[1 + \frac{x^2}{2(a+b)^2}\right] + a$$

$$= \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{a+b}\right) = \frac{x^2}{2}\left(\frac{a+b-b}{b(a+b)}\right)$$

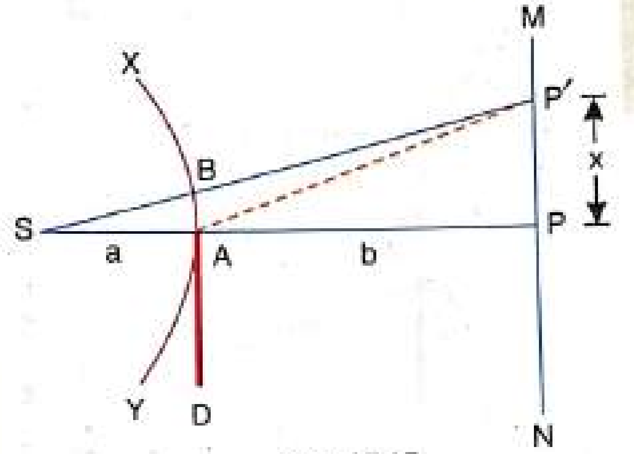

Fig. 17.17

$$\therefore \quad \delta = \frac{x^2}{2} \cdot \frac{a}{b(a+b)}$$

The point P′ will be of maximum intensity if $\delta = (2n+1)\frac{\lambda}{2}$

$$\therefore \quad (2n+1)\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)}$$

$$x_n^2 = \frac{(2n+1)(a+b)b\lambda}{a}$$

or

$$x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}} \quad (17.18)$$

where x_n is the distance of the n^{th} bright band from P.

Similarly, P′ will be of minimum intensity if $\delta = 2n\frac{\lambda}{2}$.

$$\therefore \quad 2n\frac{\lambda}{2} = \frac{ax_n^2}{2b(a+b)} \quad \text{or} \quad x_n = \frac{\sqrt{2n(a+b)b\lambda}}{a}$$

where x_n is the distance of the n^{th} dark band from P. Thus, diffraction bands of varying intensity (roughly corresponding to maxima and minima) are observed above the geometrical shadow i.e., above P and the bands disappear and uniform illumination occurs if P′ is far away from P.

17.10.2. INTENSITY AT A POINT INSIDE THE GEOMETRICAL SHADOW (STRAIGHT EDGE)

If P′ is a point below P (Fig. 17.18) and B is the new pole of the wave front with reference to the point P′, then the half period strips below B are cut off by the obstacle and only the uncovered half period strips above B will be effective in producing the illumination at P′. As P′ moves farther from P, more number of half period strips above B is also cut off and the intensity gradually falls. Thus within the geometrical shadow, the intensity gradually falls off depending on the position of P′ with respect to P.

The intensity distribution on the screen due to a straight edge is shown in Fig. 17.19. S is the source, AD is the straight edge and MN is the screen. In the illuminated portion PM, alternate bright and dark bands of gradually diminishing intensity will be observed and the intensity falls off gradually in the region of the geometrical shadow. Thus according to the wave theory, the shadows cast by obstacles in the path of light are not sharp and hence rectilinear propagation of light is only approximately true. In general, there is gradual fading of intensity in the region of the geometrical shadow and with monochromatic light bright and dark bands (diffraction bands) are observed in the

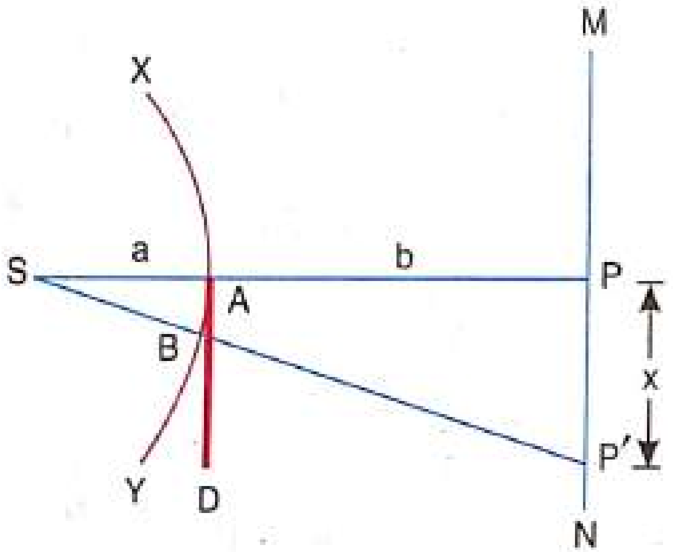

Fig. 17.18

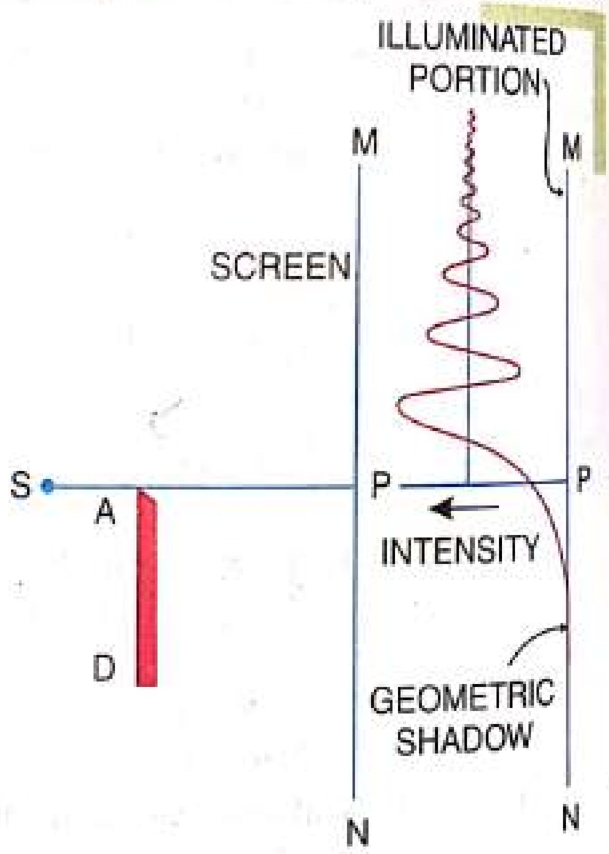

Fig. 17.19

illuminated portion of the screen. However, with white light coloured bands will be observed and the bands of shorter wavelength are nearer the point P.

14.16 Resolving power of a telescope

The purpose of a telescope is to see distant objects distinctly. The details that the device can provide *depends on the angle* subtended at the objective by two distant, close point-objects and *not on* their *linear separation.*

The **resolving power** *of a telescope is defined as the reciprocal of the smallest angle subtended at its objective by two distant close point-objects that can be* **just** *recognised as separate ones by the telescope.*

Expression for resolving power—Let AB be the objective of the telescope (Fig. 14.26) of diameter D and O_1, O_2 (not shown) be two distant close point-objects (say, two stars) from which parallel rays show an angular separation $d\theta$ at the objective. The objective (with supporting ring) acts as a *circular aperture* and *forms* Fraunhofer diffraction pattern as images of O_1 and O_2. Let P_1 and P_2 be the positions of the central maxima of the images. The patterns may be too close to distinguish them as separate. But by Rayleigh's criterion, the patterns would be *just resolved* if the central maximum of one falls on the first minimum of the other.

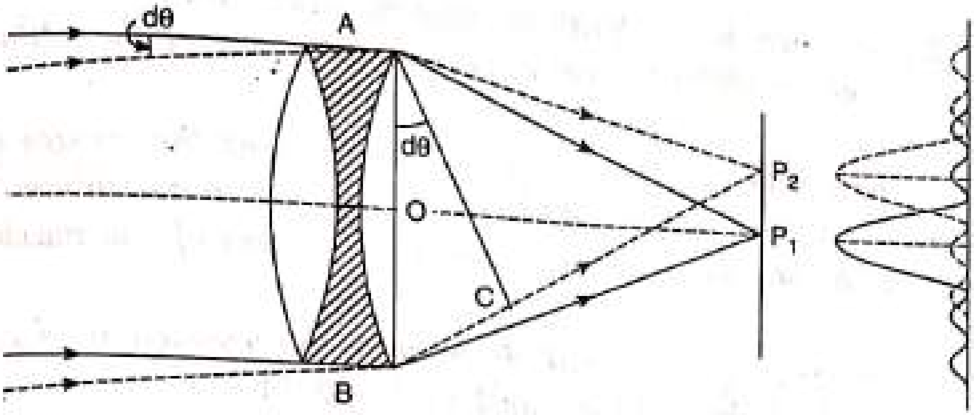

Fig. 14.26
Resolving power of a telescope

Now, the secondary wavelets travelling along AP_2 and BP_2 meet at P_2 and have a path-difference:

$$\Delta = BP_2 - AP_2 = BC$$
$$= AB \sin d\theta = AB \cdot d\theta = D \cdot d\theta$$
$$(\because \text{ angle } \theta \text{ is small})$$

If this $\Delta = \lambda$, then P_2 corresponds to the *first minimum* of the first image. So, for obtaining the ***just resolution***, we should have

$$D \cdot d\theta = \lambda \quad \Rightarrow \quad d\theta = \lambda / D \tag{14.16.1}$$

The condition holds for a *rectangular aperture*. For a ***circular aperature***, Airy modifies the condition to

$$\boxed{d\theta = \frac{1.22\lambda}{D}} \tag{14.16.2}$$

where $d\theta$ = the minimum resolvable angle between the two distant point-objects. So $d\theta$ = the ***limit of resolution***.

$$\therefore \quad \textit{Resolving power} = \frac{1}{d\theta} = \frac{D}{1.22\lambda} \tag{14.16.3}$$

The expression for the ***resolving power*** of a telescope shows that

1. A telescope with a *larger diameter of the objective* has a *higher resolving power* and conversely.

2. The *smaller the wavelength* of the light, *higher* will be *the resolving power*.

• That P_2 corresponds to the first minimum may also be understood physically. Divide the whole wavefront AB into *two* halves, AO and OB. The path-difference between the secondary waves from the corresponding points in the above two halves is $\lambda/2$. So, all the secondary waves from the two halves will interfere *destructively*. P_2 thus corresponds to the *first minimum* of the *first image*.

Magnifying power *vs.* resolving power—The *magnifying power* of a telescope should be carefully distinguished from its *resolving power*. The following features are worth-noting.

1. The magnifying power is its *ability to show an enlarged view of the object* to the eye; the resolving power, on the other hand, is its *ability to show the amount of details* or grains in the object.

2. The magnifying power *increases with distance between the centres of diffraction pattern* of two points but gives *no idea of the size of the central maximum*; the resolving power provides ideas of *both the distance between the centres of two maxima and their width.*

3. The resolving power *increases with decrease of the wavelength* of light. But the magnifying power is *independent of the wavelength* of light.

4. The magnifying power of a telescope is given by F/f, i.e., the ratio of the focal length of the objective to that of the eyepiece. *Two telescopes of the same dia for the objective* will give *different magnifying powers* (as focal length can be different), but the *same resolving power for a given wavelength* since the resolving power is given by the ratio D/λ.